This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

600048676.

CLASS-BOOK

OF

ELEMENTARY MECHANICS.

CLASS-BOOK

OF

ELEMENTARY MECHANICS.

An Introduction to Natural Philosophy.

ADAPTED TO

THE REQUIREMENTS OF THE REVISED NEW CODE.

PART II.—FORCE.

BY

WILLIAM HEWITT, B.Sc.,

SCIENCE DEMONSTRATOR FOR THE LIVERPOOL SCHOOL BOARD.

LONDON:
GEORGE PHILIP & SON, 32 FLEET STREET;
LIVERPOOL: CAXTON BUILDINGS, SOUTH JOHN STREET,
AND 49 & 51, SOUTH CASTLE STREET.
1882.

18603. e 1.

GODLEIAN LIBRARY
26 MAR 34
OXFORD.

PREFACE.

THE following lessons deal with the subjects comprised in the second and third stages of Mechanics, as defined in the New Code. Lessons 1 to 27 inclusive, on Force and Energy, treat of the subjects included in the second stage; and the Examination Questions following Lesson 27 bear on those subjects. The remaining lessons are mainly devoted to the mechanical powers, the principal subject of the third stage; and a large number of exercises and problems are given at the end of the several lessons. The Examination Questions at the end of the volume bear upon the subjects of the third stage. More than 450 exercises altogether are given in this volume, a large number of which are original, the others being for the most part selected from various examination papers.

The complete work, consisting of this and the preceding volume, will, it is hoped, be found to provide a sound though elementary introduction to the study of Natural Philosophy.

W. H.

CONTENTS.

LESSON PAGE
1. MATTER AND FORCE - 1
2. THE CHIEF FORCES OF NATURE - 4
3. THE FIRST LAW OF MOTION - 7
4. INERTIA - 10
5. FRICTION - 13
6. MASS AND MOMENTUM - 16
7. THE SECOND LAW OF MOTION - 19
8. THE COMPOSITION OF FORCES - 22
9. THE THIRD LAW OF MOTION - 25
10. THE LAWS OF MOTION - 28
11. THE FORCE OF GRAVITY - 31
12. THE LAW OF GRAVITATION - 34
13. CENTRE OF GRAVITY - 37
14. WEIGHT - 40
15. MEASURES OF FORCE - 42
16. FALLING BODIES - 45
17. THE PENDULUM - 48
18. WORK - 50
19. ENERGY - 53
20. FORMS OF ENERGY - 56
21. KINETIC ENERGY - 58
22. POTENTIAL ENERGY - 61
23. CONSERVATION OF ENERGY - 64
24. NATURAL SOURCES OF ENERGY - 67

LESSON PAGE
25. HEAT AND MECHANICAL ENERGY - - - - 70
26. THE NATURE OF HEAT - - - - - - 73
27. THE DISSIPATION OF ENERGY - - - - - 76
EXAMINATION QUESTIONS *(Lessons 1 to 27)* - - 79

28. MACHINES - - - - - - - - - 83
29. THE LEVER - - - - - - - - - 86
30. APPLICATIONS OF THE LEVER - - - - - 91
31. THE BALANCE AND STEELYARD - - - - - 94
32. THE WHEEL AND AXLE - - - - - - 96
33. THE PULLEY - - - - - - - - 101
34. THE INCLINED PLANE - - - - - - - 105
35. THE WEDGE - - - - - - - - 109
36. THE SCREW - - - - - - - - - 110
37. COMPOUND MACHINES - - - - - - - 114
38. THE HYDROSTATIC PRESS - - - - - - 117
39. THE PARALLELOGRAM OF FORCES AND VELOCITIES - 119
EXAMINATION QUESTIONS *(Lessons 28 to 39)* - - 123

CLASS-BOOK OF ELEMENTARY MECHANICS.

PART II.

LESSON 1.

MATTER AND FORCE.

Matter is the general name given to those substances with which we become acquainted by means of our senses. The various substances of which the solid ground is composed, the water of the ocean, and the air which surrounds us invisibly on all sides, are all included under the general term "matter." Notwithstanding the many various forms which matter assumes, we find when we consider their properties that they can all be arranged under three classes, known respectively as **solids, liquids**, and **gases**. Certain properties are common to all kinds of matter, such as **weight** (or more correctly gravitation), and **extension**. All the portions of matter with which we have practically to deal have also the property of **divisibility**, that is to say, they may be divided into still smaller portions. There are many reasons for believing that it would not be possible to carry this process of division of a body beyond a certain limit, without changing the nature of the body. The smallest particles of a body are called **molecules.** In solids the molecules have a definite and fixed position in the mass ; in liquids the molecules are free to move among themselves, and to change their position, without, however, leaving the mass ; in gases the molecules have no definite position, but are free to move independently of each other.

Some of the bodies that we see around us are in a state of **motion**, others in a state of **rest**. By motion we mean change of position, and we consider a body as being in motion when we find it occupying different positions at different times. If a body is observed to remain constantly in the same position we consider it as being at rest. Everything with which we are acquainted, ourselves included, is in reality in a state of motion. We are altogether unacquainted with such a state as absolute rest. When, therefore, we speak of a body as being at rest, we mean that it is at rest relatively to ourselves, or to some particular part of the earth's surface.

When we want to change a body from a state of motion to a state of rest, or from rest to motion, we find it necessary to make an effort, or, in other words, to use some **force**. To lift a stone from the

CLASS-BOOK

OF

ELEMENTARY MECHANICS.

CLASS-BOOK

OF

ELEMENTARY MECHANICS.

An Introduction to Natural Philosophy.

ADAPTED TO

THE REQUIREMENTS OF THE REVISED NEW CODE.

PART II.—FORCE.

BY

WILLIAM HEWITT, B.Sc.,

SCIENCE DEMONSTRATOR FOR THE LIVERPOOL SCHOOL BOARD.

LONDON:
GEORGE PHILIP & SON, 32 FLEET STREET;
LIVERPOOL: CAXTON BUILDINGS, SOUTH JOHN STREET,
AND 49 & 51, SOUTH CASTLE STREET.
1882.

18603 . e 1 .

to the science which studies these changes produced in the composition of bodies; and the force which brings about the changes, and which therefore gives to coal, gunpowder, &c., their power, may be called **chemical force**. And since it is a kind of attraction, or affinity, between various substances, which tends to make them unite together in the manner just described, the force is very commonly termed "chemical affinity," or "chemical attraction."

Electricity is the name given to another force which differs in many respects from the forces already enumerated. A simple illustration of electricity producing motion may be seen in the case of a stick of sealing-wax, which, after being rubbed by flannel, will attract light bodies, such as small pieces of paper, straw, &c. A piece of hot brown paper rubbed with the coat sleeve, or with a clothes brush, will produce similar effects on a somewhat larger scale. The electricity which, at a distance of fifty or a hundred miles, moves the needle of the telegraph instrument, or the plate of the telephone, is believed to be essentially the same as that obtained from the sealing-wax or brown paper, and therefore goes by the same name.

Magnetism, or the attraction exerted between a magnet and a piece of iron, is very closely connected with electricity. The most powerful magnets are simply pieces of iron, round which a wire carrying a current of electricity is wrapped several times. The force exerted between two magnets is in some positions attraction, in others repulsion. Notwithstanding the undoubted connection between electricity and magnetism, it will be convenient for us to consider the latter as a distinct force.

We must not forget to include **friction** among the important natural forces, though it is more commonly spoken of as a resistance. Its action, indeed, is usually to resist or destroy motion rather than to produce it, but the power of destroying motion is one of the characters which we have included in our definition of a force.

Capillary force, or "capillary attraction," as it is often called—the force which elevates or depresses liquids in narrow tubes or spaces, or at the points where they come in contact with solid bodies—must also be included in our list of forces.

It is very probable that, in some cases at least, forces which we have here regarded as distinct from each other may, in reality, be only different modes of action of the same force. There may also be other important natural forces which we have not included in our list. If a body were observed to move under circumstances which could not be explained as being due to the action of any of the forces we have mentioned, it would be considered as indicating the existence of some other force, possibly one previously unknown.

EXERCISES.

1. Give the names of the forces acting in each of the following cases:—
   (a) The crystallisation of water into snow.
   (b) The sinking of a ship.
   (c) The turning of a compass needle into a north and south direction.
   (d) The vibration of the prongs of a tuning fork.

2. Give some illustrations of the action of capillary force.
3. What several forces are concerned in the motion of a steam engine, and what resistances have to be overcome?
4. To the action of what forces are the motion of the wind and of a river respectively due?
5. Give some examples of gravity producing, and of its destroying, motion.
6. A piece of iron is prevented from falling, being supported by (*a*) a magnet, (*b*) a cord, (*c*) a steel spring. Name the force which resists the motion in each case.

LESSON 3.

THE FIRST LAW OF MOTION.

We have already pointed out that it is not absolutely correct to say that any body is at rest, for everything with which we are acquainted is, so far as we can ascertain, in a state of perpetual motion. For a very long time the earth was believed to be at rest, and it was not until considerable progress had been made in the science of astronomy that people began to understand that this state of rest is only apparent, and that the earth itself, with all that is on it, is in constant motion. In recent years it has been demonstrated, by more than one actual experiment, that the earth is constantly rotating on its axis, and that the rising and setting of the sun, moon, and stars, is due to this rotation. The earth is also revolving round the sun, as are also the other planets; and it has been shown that the sun itself is in motion. We may, therefore, confidently assert that we know of no body which is in a state of absolute rest; that is, which occupies exactly the same portion of space for two consecutive intervals of time. A body may be at rest with reference to the surface of the earth; a stone, for instance, may lie on precisely the same spot for any length of time. We might term such a state "relative rest," or rest in relation to the earth, or to ourselves, or to any other convenient point of reference, in order to distinguish it from "absolute rest."

It is a matter of common observation that a body at rest does not, and can not, of itself begin to move. To change its state from rest to motion, the action of some force, exerted between the body itself and some other body, is necessary. It might at first sight seem that we ourselves are exceptions to this rule: that we can set our bodies in motion by the exercise of what we have termed our vital force. But a little consideration will show that, to set ourselves in motion, we must push with our feet or some other part of our bodies against the ground, or a wall, or some other structure outside ourselves; that is to say, a force must act between our own and some other body in order that we may change our state. When we place a stone in any position, we expect to find it constantly in that position if left to itself; if it changes its position, we refer the action not to the stone, but to some force. The motion of an avalanche, or a landslip, is the result of the action of the force of gravity; the ejection of water from the geysers of Iceland is brought about by the elastic force of imprisoned steam. A body at rest, and unacted on by force, would continue for ever at rest; only the action of some external force could change its state. By "external force," we mean force exerted between the body under consideration and some other object or body external to, or outside of, the first. A force acting merely between the parts of the body itself, which we might call an internal force, could not produce motion of the body as a whole. As an illustration of this statement, we may consider the case of a man sitting in a boat at rest. By no amount of force exerted on the boat can he move it along the water. To do that he must exert a force on some object external to the boat, such as by pressing on the water with an oar, by pushing with a pole against the bed of the stream or lake, or by pulling a rope fastened to the bank, or to some vessel or other body. To move the boat from a state of rest (or, as may readily be seen, to stop it when in a state of motion) external force is required. The same is true of all bodies whatever, and we may therefore state, generally, that a body at rest will continue in that state if not acted on by some external force.

A body in motion has no more power to alter its motion, than a body at rest has to change its state of rest. A body once set in motion will continue in motion, with the direction and velocity of its motion unaltered, if removed from the action of all external force. The truth of this statement is not quite so obvious as that a body at rest cannot put itself in motion. We know by experience that a moving body may be made to move faster, or slower, or in a different direction, by applying a suitable force to it. And we also know by experience that a body moving at a certain rate never has its velocity increased unless by the action of some force. The constantly increasing velocity of a falling body is due to the continued action of the force of gravity, just as the increasing speed of a train leaving a station is due to the continued action of the steam on the pistons connected with the wheels. A little careful observation will, in like manner, satisfy us that the constantly decreasing velocity of a body, such as a train entering a station, or a stone rising into the air, is due to continued action of some resisting force (friction, &c.), and that the more we diminish such resistance the slower is the rate at which the velocity of the body decreases. If a top is set spinning, it will continue to spin for a longer or shorter time, in spite of the resistance of the air and the friction of its point against the ground. If the top be made in the form of a heavy disc, so as to present only a small surface to the air, and if its point be made of smooth hard steel, and rest on a smooth glass surface, the motion will last for a very much longer time than under ordinary circumstances. Further, if the top be spun in a vacuum under an air-pump, so as to remove, at any rate to a very large extent, the resistance of the air, the rate at which the velocity diminishes will be still further reduced, and the motion will continue for a very long time, and will be nearly uniform from instant to instant. It appears, therefore, that the decrease of the velocity is due to the action of external resistances, and that as they are gradually diminished the velocity tends more and more to become uniform. We have but to go one step farther to arrive at the conclusion, that if we could only remove all resistance to the motion of the top, *i.e.*, could prevent all external force from acting on it, it would never stop, but would continue to move for ever with a uniform velocity. It is impossible in practice to remove all resistances to the motion of bodies, so that we cannot actually point to any body moving uniformly in the absence of external force. The motion of the earth round the sun is perhaps the best example we can give of an approach to this state of perpetual motion. But if, as is probable, there is a resistance, however small, to the motion of the earth, that motion cannot continue uniform. We are now, therefore, in a position to say that the velocity of a moving body will remain uniform unless the body be acted on by external force; in other words, external force is required to change the velocity of a body.

Again, on a piece of smooth and level ice, a ball set rolling in any direction moves in a straight line, turning neither to the right hand nor the left, unless made to do so by some irregularity in the surface of the ice, some crack, or slope, or obstacle against which it strikes. On a smooth and level bowling-green, the player expects his ball to travel in a straight line in the direction in which he projects it. A stone or other body falling freely from a height, moves in a straight vertical line, and deviates only when acted on by the wind, or by striking against some obstacle. And we may state, generally, that every body in motion tends to move in a straight line, and changes its direction only under the action of external forces. It appears, therefore, that the direction, as well as the velocity, of the motion of a body not acted on by external force, will be constant.

There is, then, no power residing in a body which enables it of itself to change its state, whether that state be one of motion or of rest. Every change which occurs in the state of a body is due to the action of some force external to the body; or, more correctly, to some force exerted between the body itself and some other body. Any change in a body's state of rest or motion is an evidence of the action of some force, and we might define force as any cause which produces a change in the state of a body.

This most important proposition which we have been considering was given by Sir Isaac Newton as the first of his famous three "laws of motion." His book, called the "Mathematical Principles of Natural Philosophy," (usually called the "Principia," or principles, being written in Latin), written 200 years ago, in which these laws are contained, is considered to be one of the most important books ever written. His statement of the first law of motion may be thus translated :—**Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it is made to change that state by external forces acting upon it.**

EXERCISES.

1. A stone is rolling down a hill: what forces tend to keep it in motion, and what to stop it?
2. A pendulum set swinging, and left to itself, ultimately comes to rest: point out the resistances which have destroyed its motion.
3. What force propels a rifle bullet out of the gun? If the gun is held in a horizontal position whilst being fired, what causes the bullet to deviate from a horizontal straight line in its course?
4. What resistances are there to the motion of a train? Explain, as far as you can, why the speed gradually increases as the train starts, and gradually decreases as it stops.
5. Compare the motion of a stone thrown vertically upwards, with that of the train in the last question.
6. Why is a bowling green made as level as possible, the grass closely cut, and the bowls of smooth polished wood?

LESSON 4.

INERTIA.

The first law of motion, as explained in the preceding lesson, asserts that, except by the action of external force, a body never changes its state of rest, or of uniform motion in a straight line. When we apply a force to a body in order to change its state, we become aware of certain effects which we will now briefly study. In the first place, whenever we attempt to set a heavy body in motion, we experience more or less resistance on the part of the body; and this is especially the case when the force is applied suddenly. In many cases this resistance is due in part to friction, but not altogether. For if the body be suspended by a cord, so as to move with scarcely any friction, there will still be a considerable resistance opposed to a force tending to suddenly move the body. If a body so suspended is pulled sideways by means of an attached piece of indiarubber, the indiarubber will be seen to stretch before the body moves; and the amount of elongation will be greater the more suddenly the force acts. A piece of thread similarly attached might be readily broken by a sudden snatch. The stretching of the indiarubber, and the breaking of the thread, both prove the existence of a resistance opposed by the body to the force tending to set it in motion.

Again, if a body suspended from a piece of indiarubber, or from a steel spring, is slightly raised, and allowed to fall, it will be observed that the indiarubber or spring is stretched more than when the body hangs quietly at rest from it. The velocity which the body has acquired in falling cannot be suddenly destroyed, the moving body offers a resistance to the force tending to bring it to rest. In the same way a person cannot all at once stop a boat or railway carriage in motion; there is a resistance offered by the moving body which must be gradually overcome. It appears, therefore, that an external force, tending to change the state of a body, has to overcome a certain resistance on the part of the body itself, and to this resistance the name of **inertia** is given. **Inertia is the resistance which a body offers to any force which tends to change its state of rest or motion.** Inertia is a universal property of matter, and in fact is the characteristic property of matter. To say that a body has inertia is merely to assert the first law of motion in another form, viz., that expenditure of force is necessary to produce a change in the state of a body.

The effects of the inertia of bodies are very common and familiar, as the following selected examples will show. Every one is familiar with the fact that if a vessel containing a liquid, *e.g.*, a cup containing tea, or a dish containing gravy, be suddenly moved sideways, the liquid leaves the forward side of the dish, and rises up on the hinder side, perhaps so far as to fall over and be spilled. The liquid at rest opposes a resistance to the force tending to set it in motion, and if the vessel is moved with sufficient rapidity, the resistance may hold back the liquid so long that the vessel is moved from under it. If, after the vessel has been set in motion, it is suddenly stopped, a movement of the liquid in the opposite direction to that in the former case is observed. The inertia of the liquid keeps it moving forward after the containing vessel has been stopped, and possibly may carry it over the front side of the vessel.

A person sitting on horseback, or in a carriage, involuntarily leans backward when the horse or carriage begins to move forward, and the backward movement is the more marked the more suddenly the horse or carriage starts forward. If the horse or carriage when in motion is suddenly stopped by any means, the inertia of the rider may be so great as to throw him forward with great force, and this is a frequent cause of serious accidents.

A boy wishing to leap over a brook, usually "takes a run" from some little distance to the edge of the brook, and then gives his spring. The inertia of motion due to the run is added to the force of the spring, and assists in carrying his body over the brook. On the other hand, if, while a boy is running along quickly, he catches his foot against a stone, or other obstacle, he is thrown down forwards. The motion of his feet has been suddenly arrested, but the inertia of the rest of his body carries it forward, and he falls down, just as if while standing his feet had been suddenly pulled from under him.

The effect of inertia is also seen in the swinging of a pendulum, after it has been displaced from its position of rest. The pendulum not only falls down to its lowest point, but the inertia of the motion thus acquired carries it forwards, and raises it on the other side. In the same manner, if the string attached to a slate be twisted, and the slate be allowed to move so as to untwist the string, it will not come to a dead stop as soon as the last turn of the twist is undone. By that time the slate will probably have acquired a considerable velocity, and, by virtue of its inertia, will continue to move in the same direction, twisting the string in the direction opposite to that of the original twist. The inertia of the rotating slate will keep it moving until its motion is gradually destroyed by the resistance of the air and the elasticity of the twisting string.

It has been observed that where a river changes its course, the bank on the outer side of the bend is always worn away faster, and undermined more, than that on the inner side of the curve. This is easily explained by the fact that the tendency of the water is (according to the first law of motion) to continue moving in a straight line. It is, therefore, projected with some force against that bank (*b*, fig. 1) which tends to change its direction. A railway train passing round a curve similarly presses with greater force against the outer rail of the curve than against the inner. The tendency of the train to continue moving in a straight line shows itself as a force tending to throw it off the rails on the outer side of the curve, and this is counteracted by making the outer rail slightly higher than the inner. The difficulty which a person experiences in suddenly turning aside when running quickly, is due to this same tendency to move always in a straight line, as is also the force with which a stone in a sling pulls against the hand. The tendency which a body moving in a circular path has to move farther away from the centre of the circle, and continue its motion in a straight line, is called its **centrifugal tendency,** or sometimes "centrifugal force." It is, of course, due simply to the inertia of the body. The fly-wheel, seen in many engines, owes its efficiency to the principle of inertia. Any obstruction tending to stop the engine has to overcome the inertia of the heavy and rapidly-moving fly-wheel, in addition to the force of the steam or motive-power at that instant. The fly-wheel, therefore, assists the engine in overcoming any small obstructions, which would otherwise produce irregularities in the working of the engine, and thus it tends to keep the motion of the engine uniform.

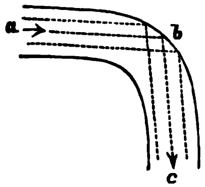

Fig. 1.

The examples we have now considered are but a small proportion of those which might be given to illustrate the effects of inertia. In every case where a force tends to set a body in motion, or to change the direction or velocity of the motion, this resistance has to be overcome. The amount of inertia varies very considerably in different cases. Even with the same body the inertia is considerably greater when the force is suddenly applied than when it is applied more gradually. If a body is perfectly free to move, the smallest possible force will be sufficient to set it in motion. But to produce a certain velocity it must act on the body for a longer time than would be necessary with a larger force. To set a body in motion in an infinitely short space of time, an infinitely great force would be required. To produce a certain change in the state of a body there is a certain definite resistance to be overcome, and this may be done either by a large force acting for a short space of time, or by a smaller force acting for a proportionally longer time. The intensity of the resistance offered by a body depends, therefore, on the manner in which the force is applied, whether gradually or suddenly. It depends, also, upon the quantity of matter contained in the body, the mass of the body, as we shall hereafter learn to call it. The greater the mass of a body (or, what comes to practically the same thing, the heavier the body) the greater is its inertia.

EXERCISES.

1. Give some illustrations of the inertia of air.
2. Show that in the case of one person suddenly knocking against another, it is on account of their inertia that both are hurt.
3. Why does a cricketer move his hands backwards when catching a swiftly moving ball?
4. Show that the power to throw a stone or a ball is due to inertia.
5. Explain the common method of fastening on the head of a hammer, by holding it upright and knocking the handle sharply against the ground.
6. Explain why a person stepping from a train or carriage in motion is so liable to be thrown down.
7. When a cup of tea is stirred, the liquid partly leaves the middle, and heaps itself up against the sides of the cup. Explain this.

LESSON 5.

FRICTION.

We have pointed out that the assertion in the first law of motion, that moving bodies are powerless to stop themselves, is less obvious than the assertion that bodies at rest cannot set themselves in motion. All our experience goes to show that bodies, when set in motion and left to themselves, very soon come to rest. This, however, is due, as we have explained, to the action of resisting forces, one of the most important and familiar of which is friction. We will devote the present lesson to a further consideration of this resistance.

Friction is the name given to the resistance which a body encounters when moving in contact with the surface of another body, as when a box is pushed along the ground, or a boat is pulled through the water. That the resistance is due in some manner to the contact of the two surfaces is very obvious; and that its amount depends upon the nature and condition of those surfaces is also a matter of common experience. We speak of bodies as **rough** or **smooth**, but these terms are merely relative; that is to say, one body is smooth only as compared with another. We have no experience of such a condition as absolute smoothness. The smoothest surface which we can produce on a body is still very uneven, and a good microscope will usually show that the points forming the surface are at very different levels; so that on a small scale the surface presents innumerable irregularities. In neither the smoothest ice nor the most highly polished glass do the points forming the surface lie absolutely in one plane.

We may, therefore, readily conceive that, when two such surfaces are moved in contact, the irregularities on the one surface will catch against those on the other, and thus a resistance to the motion will arise. We may roughly illustrate this by reference to the case of two brushes moved over each other, and at the same time pressed slightly together. The bristles of the one brush catch against those of the other, and the motion is thus resisted. The greater the force with which the brushes are pressed together while being moved, the greater is the resistance. And in the case of the friction between any two bodies, its amount depends upon the pressure forcing the bodies together. It has been proved by experiment that the amount of friction between two bodies is proportional to the pressure; so that if in any case we double the pressure, without altering any of the other conditions, the amount of friction is doubled. The amount of friction has been found by experiment not to vary with the extent of the surfaces in contact, so long as the pressure remains the same, and to be independent, also, of the velocity with which the body moves. For a given pressure, the amount of friction varies considerably according to the nature of the surfaces in contact, both as regards the material of which they are formed, and the degree of smoothness and polish given to them. As a general rule, there is most friction between soft bodies, and least between hard bodies. Thus, to move one smooth piece of wood over another smooth piece of wood, the amount of friction to be overcome has been experimentally found to be equal to about one-half of the weight of the piece moved, though it becomes somewhat less as the surfaces of the wood are more and more worn by use. To move one smooth piece of iron over another, the amount of resistance due to friction is found to be equal to only about one-fifth part of the weight. Thus a force of 10 lbs. would move a piece of iron weighing 50 lbs. over a surface of iron, both being supposed to be smooth; whilst it would require a force of 25 lbs. to move a piece of wood weighing 50 lbs. over a surface of wood.

Friction not only acts to resist the motion of a body already moving, but also to prevent a body at rest from being set in motion. And for the same bodies the friction in the latter case is somewhat greater than in the former, and seems to depend to some extent on the time during which the surfaces have been in contact. A brick will rest on an inclined board until the inclination reaches a certain angle, when the brick begins to slide down. At a smaller inclination, however, the brick will continue to slide if it be gently started, thus illustrating the above statement, that the friction which tends to prevent a body at rest being set in motion is greater than the friction acting on the same body when in motion.

It is usual to distinguish two varieties of friction, called respectively **sliding friction**, and **rolling friction.** Sliding friction is encountered when one body slides over the surface of another, and is the variety we have considered in the preceding paragraphs; whereas rolling friction is met with when one body rolls over another. The difference between these two varieties of friction is well illustrated by the difference between the motion of a carriage and of a sleigh. It is well known that, for the same kind of surfaces, rolling friction is considerably less than sliding friction. This will explain the use of rollers or castors on tables, pianos, and other articles of furniture, wheels on carriages, and wooden rollers under heavy blocks of stone which have to be moved.

In a large number of cases we must look upon friction as a hindrance—a resistance involving a considerable waste of power, and something, therefore, to be diminished as far as practicable. Fortunately there are several means by which the amount of friction between bodies may be considerably reduced. In the first place, it is obvious that the surfaces in contact should be made as hard and smooth as possible. Metals moving on metals should be employed rather than wood on wood. The axles of wheels, and the sockets in which they turn, should be made as truly cylindrical, and be as highly polished, as possible. They should also obviously be constructed of those materials which are found by experiment to produce the least friction when moved in contact with each other. In the second place, it is found that certain substances (known as lubricants, or lubricating substances) interposed between the rubbing surfaces diminish the friction considerably. Such are various kinds of grease and oil, and black-lead or graphite. And, thirdly, it is an advantage to substitute, when possible, rolling for sliding friction; as by mounting the body on wheels or rollers, or making it of such a shape that it can be rolled over and over. If the object be to prevent or to check motion, then, of course, sliding friction is employed, as in the case of locking a cart wheel before descending a hill, and in the case of brakes generally.

It is well that we should remember that friction does not always act to our disadvantage. A little reflection will convince us that, in the absence of friction, it would be impossible to do many things which we now do quite easily. Without friction we could not, for example, hold a pen or pencil with which to write; and no body could rest without support on a surface which deviated in the least from a horizontal position. A horse would be unable to draw a cart, and the wheels of a locomotive engine would simply turn round without advancing, just as we may sometimes see them do when the rails are slippery. It would be a difficult matter for us to set ourselves in motion, and, once in motion, it would be as difficult for us to stop. Nails and screws would be useless, and it is very doubtful whether the walls of our houses would be able to stand. From our experience in attempting to walk about on smooth ice, which is far from being altogether without friction, we may gather some slight

idea of the difficulties in which we should find ourselves were friction to cease altogether to act.

EXERCISES.

1. What part of a river would you expect to find in most rapid motion, and why?
2. Explain why it would be impossible for a horse to draw a cart along a frictionless road.
3. Mention some instances where means are taken to increase friction, and others where it is diminished as much as possible.
4. Give several examples of sliding and of rolling friction respectively.
5. The amount of friction resisting the motion of a carriage on an ordinary road is equal to about $\frac{1}{15}$th part of the weight moved, and on a railway to about $\frac{1}{300}$th part. Find what force will have to be exerted in each case to draw a load of 9 tons.
6. Why is it more difficult for a horse to start a heavily loaded cart than to keep it in motion when once started?

LESSON 6.

MASS AND MOMENTUM.

By the weight of a body we mean a certain pressure towards the earth which the body exerts in consequence of the action of gravity. We are so familiar by experience with the fact that all bodies possess more or less weight, that it is difficult for us to conceive of a body without weight. But it is certain that in the absence of gravity bodies would have no weight; and it is also certain that if the intensity or strength of gravity varied from time to time, so also would the weight of every body vary. We shall have an opportunity in a future lesson of considering, more fully than we could now do, the action of gravity, and the effects it produces, amongst which effects is that of weight. What we want to insist on at present is the fact that weight is a property of matter which depends upon the action of gravity. There are, however, other properties of matter which are not dependent upon gravity, and which would remain unaltered even if gravity were altogether destroyed. A body, for example, contains a certain definite quantity of matter, and the weight of the body depends upon the quantity of matter it contains, as well as on the intensity of gravity at the place. But though the amount of gravity acting on the body depends upon the quantity of matter of which it is composed, the quantity of matter does not depend upon the force of gravity. If the body were carried from place to place on the earth's surface, or projected to a considerable distance above the surface of the earth, its weight would change, because the intensity of gravity is not quite the same in these different places. But the quantity of matter would remain absolutely the same in every place, because it is altogether independent of gravity. A body free to move opposes the same amount of resistance (inertia) to a force tending to alter its state in every place, however great or small the force of gravity at the place may be. Inertia in fact does not depend upon gravity, and would be present to the same extent in the absence of gravity. But it does depend upon the quantity of matter contained in a body, the greater the quantity of matter the greater the inertia. The weight of a body might change, or be altogether lost, but its inertia would remain the same; and it is therefore not strictly correct to say that the inertia of a body depends upon its weight. And in many other cases it is plain that it is the quantity of matter in a body, and not the weight of the body at that particular place, which influences the results observed. As a matter of convenience, it is usual to employ the term **mass** in place of the expression "quantity of matter;" and to speak of the mass of one body being equal to, or greater or less than, the mass of another body. It is very important that we should have a clear comprehension of the exact difference between mass and weight. The word mass should suggest to our minds simply the idea of a certain quantity of matter, without reference to any particular place or time, or to the action of gravity or any other force; while with weight we must connect the idea of the action of the force of gravity, and its variation in amount as gravity varies.

It is necessary that we should have some means of ascertaining the quantity of matter in a body, so far at least as will enable us to compare the mass of one body with that of another, and with some chosen standard. This we are able to do in consequence of the fact, which has been ascertained by experiment, that equal forces similarly applied to equal masses of matter produce equal effects, whilst if the masses on which they act are unequal, the effects are also unequal. The effect of gravity, for instance, on two pints of water is twice as great as its effect on one pint, as is shown by the difference in the weight. Four cubic inches of lead weigh four times as much as one cubic inch of lead weighs. If we strike a small stone with a stick, and so impart to it a certain velocity of motion, and then exert a precisely similar and equal force on a much larger stone, *i.e.*, a stone having a greater mass, the effect produced, as measured by the velocity imparted, is less than before. If the mass of the one stone is twice that of the other, the velocity imparted to the latter will be twice as great as that imparted to the former. Suppose that two cannon balls are fired, under precisely similar circumstances, by the explosion of equal quantities of the same gunpowder, and that the velocity imparted to one ball is three times as great as that imparted to the other. We should in that case conclude that the mass of the one body is three times as great as the mass of the other, the body started with the smaller velocity being the one containing the greater amount of matter.

The relative masses of different bodies may therefore be estimated by observing and measuring the effects produced by the action of equal forces upon the bodies. As a general rule, the force whose effects are observed and measured is gravity, since it is universally present, easily applied, and the effect produced—weight—is readily measured. If a number of bodies are weighed at the same place, or at places where the intensity of gravity is the same, the ratio between the weights of any two of them will indicate the rate existing between their masses. A body weighing 10 lbs. contains ten times the quantity of matter which is contained in a body weighing 1 lb.; and this is true whatever the kind of matter composing the bodies may be, provided only that suitable precautions have been taken to ascertain the true weight.

The standard or **unit of mass** usually adopted in England is the quantity of matter contained in the piece of platinum, which Parliament has decreed shall be considered the standard pound weight. The pound weights in ordinary use are supposed to have exactly the same weight as this standard; and if accurately made they contain a quantity of matter equal to that in the standard. A body of any kind weighing one pound contains one unit of mass, a body weighing seven pounds contains seven units of mass, and so on. The ordinary weighing scales are instruments for determining the masses of bodies, and the process of weighing consists in measuring out a quantity of one substance having a mass equal to that of the weights in the other scale.

Momentum is a term frequently employed in Mechanics which we are now in a position to explain. If two bodies having unequal masses are moving with equal velocities, it will require more force to stop the one with the larger mass than to stop the other. The one having the larger mass would have the greater amount of moving force, and would be able to overcome more resistance before being brought to rest than the other one could overcome. Momentum is the term used to express the **quantity of motion,** or amount of moving force, possessed by a body in motion. It depends upon two things—the quantity of matter, or mass, of the body, and the velocity; and is expressed numerically by multiplying together the numbers representing the mass and the velocity respectively. Thus, a body having a mass of 10 units, and a velocity of 15 units (15 feet per second), would be said to have a momentum of 150. A large iceberg, with its enormous mass of ice above and below the water level, may be moving with a velocity scarcely perceptible, yet its momentum is so great that a ship caught between it and another similar mass would be crushed as easily as we might crush a cardboard box. On the other hand, a rifle bullet, with its very small mass but great velocity, has sufficient momentum to enable it to overcome the resistance of the air through a distance of more than a mile, or to penetrate a considerable thickness of hard wood. The effect of a high velocity, combined with a considerable mass, is well illustrated by the case of a modern cannon ball, which, when weighing 800 lbs., and moving with a velocity of 1,400 feet per second, has been known to penetrate an iron plate more than 19 inches thick.

EXERCISES.

1. A certain force acting for one second on a body weighing 28 lbs. produces a velocity of 10 feet per second; what velocity will it produce in a body weighing 1 cwt.?
2. The air exerts buoyancy or upward pressure on bodies in it similar to that exerted by liquids, but much smaller in amount. If a piece of cork on one scale-pan balances a piece of iron on the other, is the mass of the cork equal to that of the iron? Give reasons for your answer.
3. A man can row a boat with a velocity of 4 miles an hour, the boat and man together weighing 2 cwt. A load weighing 56 lbs. is taken into the boat; what will be the velocity, the man rowing as before?
4. Why is it an advantage to make the head of a sledge-hammer as heavy, and the handle as light, as possible?
5. If a quantity of sand in one scale just balanced a piece of iron in the other, would the balance be maintained if the scales were carried to a place where the intensity of gravity was somewhat different,—for example, down a deep mine? Give reasons for your answer.
6. If a cannon weighing 10 tons starts backwards when fired with a velocity of 25 feet per second, and the ball weighing 280 lbs. moves forwards with the same momentum, find the velocity of the ball.

LESSON 7.

THE SECOND LAW OF MOTION.

The first law of motion treats of the condition of bodies not acted on by external force; the second law of motion, on the contrary, deals with the case of bodies subject to the action of such forces. And in the first place it implies, though it does not directly state, that it is impossible for a force to act upon a body without producing an effect. However large, or however small, a body may be, whatever its state of rest or motion, the application of a force to the body involves of necessity a change in its state. But this law of motion goes farther, and teaches that the change produced by a certain force is precisely the same in every case, the magnitude and direction of the change being determined only by the magnitude and direction of the force. The consideration of one or two examples will perhaps best serve to make clear this most important principle.

Take first the case of a vessel drifting down a river, impelled only by the force of the current. It will be moving under the action of this force in a certain definite direction, and with a definite velocity. Suppose that now a wind commences to blow, and that its force acts on the vessel in addition to the force of the current. The motion of the vessel cannot possibly be precisely the same as in the first case; the direction, or velocity, or perhaps both direction and velocity, will be changed. The direction and magnitude of the change will depend upon the direction and magnitude of the force of the wind. Suppose the force of the stream alone to be able to carry the vessel along at the rate of three miles an hour, and the force of the wind alone to be able to move it at the rate of four miles an hour. If both forces act together in the same direction, the vessel will move with a velocity of seven miles an hour, the change of motion produced by the added force of the wind being exactly proportional to the magnitude of that force. If the direction in which the force of the wind acts is exactly opposite to that of the current, the vessel will move against the current with a velocity of one mile an hour. It is obvious that both forces are in action in this latter case, and that they are each producing precisely the same effect as in the former case; that is to say, the force of the stream is producing a velocity of three miles an hour, and the force of the wind a velocity of four miles an hour. But as the velocities are in opposite directions, the effect of one must be subtracted from that of the other, in order to ascertain the resultant effect on the body. The main point to be understood is that, notwithstanding the great difference in the actual velocity of the body in the two cases, the effect of each force is precisely the same in each case. Now, once more suppose the direction of the wind to be inclined to that of the stream, say at right angles; the effect of the wind would be to carry the vessel across the stream. But this would not interfere with the motion of the vessel down the stream, the velocity of which would remain just the same as before the wind commenced to act. Let us represent the motion of the vessel on a small scale by the diagram (fig. 2). The vessel, which we will suppose to be at A, is acted on by the force of the stream in such a manner as would carry it through three spaces (say 3 miles) to B, in a certain time, while the force of the wind acting alone for the same length of time would carry it from A to C, through four spaces. Experiment shows that, when both forces act together, the vessel will move from A to D, in the time during which one force would carry it from A to B, or the other from A to C, if acting singly. The case is stated generally by Newton, thus:—"A body by two forces conjoined will describe the diagonal of a parallelogram in the same time that it would describe the sides by those forces apart." It is plain that the motion of the vessel along this diagonal is due to neither force singly, but to the two conjoined, and also that each force produces precisely the same effect as if it acted alone. For the vessel actually moves down the river a distance equal to A B, while at the same time it goes across the river a distance equal to A C.

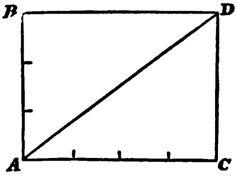

Fig. 2.

Suppose a man sitting in a railway carriage, which is at rest, throws a ball to strike the top of the carriage; if, when the carriage is in rapid motion, the man wishes to throw the ball to strike the same place as before, it will be sufficient to use a force precisely similar to that which he used in the first instance. The fact that the carriage and person and ball are moving makes no difference in the result; a force applied to the ball has precisely the same effect to produce motion as if all were at rest. A penny rolling off the seat of the carriage would reach the floor in exactly the same time whether the carriage were at rest or in motion. When we wish to set a body in motion in a certain direction, or to change the velocity or direction of a body in motion, we do not stop to consider how many forces may already be acting on the body, or how it may be moving round the earth's axis, round the sun, &c. We apply our force quite independently of any others that may be acting on the body, and expect to see a change produced in the state of the body proportional to the amount, and in the direction of, the force employed. The previous state of the body need not be taken into account; any force applied to the body will produce a velocity in a certain direction, and the direction and amount of velocity produced will be exactly the same if the body is already in motion as if it were at rest.

We may render Newton's statement of the second law of motion as follows:—**change of motion is proportional to the force applied, and takes place in the direction in which the force acts.** It will be observed that nothing is said about the previous state of the body to which the force is applied, because, as we have just pointed out, the change of motion produced is independent of the previous state of the body. By "change of motion" in the law we must understand change of quantity of motion, or momentum; that is to say, in estimating the effect of a force we must take into consideration the mass of the body moved, as well as the velocity. A certain force acting on a body with a mass of 10, will produce a velocity twice as great as when acting on a mass of 20, but the momentum would be the same in each case. It will be evident that if we acknowledge the truth of this law of motion, it furnishes us with a means of measuring the relative strengths of various forces. The relative strengths of two forces will be in the proportion of the effects they can produce. A force which can produce a change of momentum equal to 100, must have double the strength of a force which, in the same time, can produce only a change of momentum equal to 50.

Numerous examples might be given in illustration of this important law of motion; each proving that the actual direction and velocity of the motion of a body is the result of the action of the whole of the forces applied to the body, and that were one force, however small, removed, or an additional force added, the motion of the body would be changed. However many forces might be acting on the body, each would be producing the same effect in the direction of its action as if it alone were acting on the body. We will, however, refer to one more illustration only. A cannon ball lifted to the mouth of the cannon, and allowed to fall, would reach the ground in a certain time, depending on the height from which it fell. If put into the cannon and fired horizontally, the ball would fall to the ground after it left the cannon in precisely the same time as before. The horizontal velocity, imparted to it by the force of the explosion, would not interfere with the vertical velocity caused by gravity. If the ball is required to travel a long distance horizontally, the mouth of the cannon is elevated, so that the horizontal motion may continue both while the ball is rising into the air, and while it is falling from the height to which it rises.

EXERCISES.

1. Explain why a ball which a boy throws straight up while running, falls into his hands again.
2. Why is a collision between two trains moving in opposite directions, more disastrous than if they had been moving in the same direction, with the same velocities as before?
3. A person drops a heavy ball out of the window of a railway carriage in rapid motion; where would the ball fall, if there were no resistance due to the air?
4. A ship is sailing westwards with a velocity of 1,000 feet per minute, and a man on deck walks backwards and forwards, from stern to bow, at the rate of 400 feet per minute; find his actual velocity, (*a*) when moving towards the stern, (*b*) when moving towards the bow.
5. If in the last question the man walks across the deck from side to side, with the same velocity as before, show by a diagram the direction in which he will be actually moving. With what velocity will he be moving westwards in this case?
6. A rider in a circus sometimes juggles with balls while riding in a standing position on horseback. For what reason is the exercise more difficult than when he stands on the ground?

LESSON 8.

THE COMPOSITION OF FORCES.

In order to describe fully the motion of a body, it is necessary to state the **direction** in which, and the **velocity** with which, the motion takes place. In order to describe a force, it is necessary to specify three elements or particulars concerning the force—viz., the **magnitude** or amount of the force, the **direction** in which it acts, and the **point of application**, or point on which it acts.

It is possible, and in many cases very convenient, to represent a force, and a velocity or motion, by means of a straight line. Thus in figure 3, the line A B may indicate the velocity of a body as being in the direction of the line from A to B, and equal in amount to five units of a given scale previously arranged. If each unit, for instance, represented one mile per hour, the velocity indicated by A B would be five miles per hour. The direction may be indicated by affixing an arrow head, as in the figure, or simply by the order in which the letters are used. The same line A B might be taken to represent a

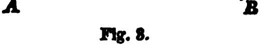

Fig. 3.

force, its point of application being represented by A, its direction by the direction of the line A B, and its magnitude five units according to the scale, *e.g.*, 5 lbs. or 5 tons. In the following paragraphs we are about to consider some important rules in connection with the simultaneous action of two or more forces on a body, and we shall find it convenient to represent the forces under consideration by lines, according to the principle just explained. The rules we shall consider follow directly from the second law of motion, which was explained in the last lesson.

When two or more forces act on a body at the same time, without producing motion, the body is said to be in **equilibrium.** A weight supported in the hand is in equilibrium under the action of gravity and the upward force of the hand, the effect of the one balancing the effect of the other. When several forces act together on a body, and do not produce equilibrium, the result is that the body moves in some definite direction, and with a definite velocity. In almost every case it would be possible to produce the same effect by the action of a single force. The single force which could replace the several forces acting in any case, and produce the same effect, is called the **resultant** of those forces. And the process by means of which the resultant of two or more forces is determined is known as **the composition of forces.**

To find the resultant of a number of forces acting in the same straight line, it must be noticed whether all act in the same direction, or whether the direction of some is opposite to that of the others. In the former case the resultant will be equal in magnitude to the sum of the several forces, and will act in the same direction. In the latter case, the magnitude of the resultant will be equal to the difference between the sum of the forces acting in one direction, and the sum of those acting in the other direction; and the direction of the resultant will be the same as that of the forces whose sum is the greater. Thus in fig. 4, the two forces C A, A B, the magnitudes of which are respectively 4, and 3, are represented as acting in the same direction on point A. The same effect would evidently be produced by a single force, A R, having a magnitude of seven, and acting in the same direction as the two original forces. A R, is therefore the resultant of C A, A B.

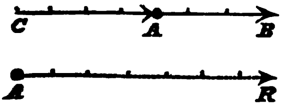

Fig. 4.

In fig. 5, two forces, A C, A B, are represented as acting on a point A, in opposite directions, and having magnitudes of four and three respectively. The point A, if free to move, would move in the direction of the larger force A C, but with a velocity equal only to that produced by a single force having a magnitude of one unit. A R might there

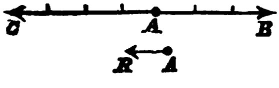

Fig. 5.

fore replace the two forces A C, A B, and produce the same effect on the point; it is therefore the resultant of those forces.

If the forces are parallel to each other instead of being in the same straight line, the rules for finding the direction and magnitude of the resultant are similar to those given above. That is to say, the resultant is equal to the sum, and acts towards the same direction, when the parallel forces act in the same direction as each other; while it is equal to their difference, and acts in the direction of the greater, when the original forces act in opposite directions. The rule for finding the point at which the resultant must act is, however, not so simple, and cannot be explained here; while in some cases it is not possible to have a single resultant. The particular case of the action of parallel forces which will most directly affect us is that of the weight of a body, as will be explained in a subsequent lesson. In fig. 6, four parallel forces, A*a*, B*b*, C*c*, D*d*, are shown, all equal, and acting in the same direction. The resultant R*r*, in this case, will be a force equal to the sum of the four forces, and acting in the same direction, at the middle point of the line on which the forces may be considered as acting.

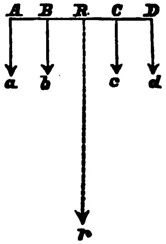

Fig. 6.

When two forces act on a point in directions inclined to each other, the magnitude and direction of the resultant are found by a simple geometrical construction. The rule for finding the resultant is exactly the same as, and is in fact derived from, the rule for determining the amount and direction of the resultant velocity of a body acted on at the same time by two forces producing velocities in different directions. This was explained in the last lesson, and is known as the **parallelogram of velocities.** The rule for finding the resultant of two inclined forces acting on a point is known as the **parallelogram of forces.** It may be stated as follows:—Take two straight lines drawn from a point to represent the two forces in magnitude and direction; complete the parallelogram of which these two lines form adjacent sides, and draw the diagonal from the point of application of the forces to the opposite corner; this diagonal will represent the resultant in magnitude and direction.

Thus, in fig. 7, let AB, AC, represent the direction and magnitude of two forces acting at A, the direction and magnitude of their resultant will be represented on the same scale by AR, the diagonal of the parallelogram of which the lines AB, AC, form two sides. That is to say, a single force represented by AR, would move, or tend to move, the point A, exactly the same as the two forces represented by AB,

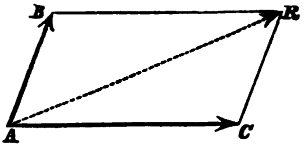

Fig. 7.

AC, acting together. And a force equal in magnitude to that represented by AR, but acting in the opposite direction, would just be sufficient to balance the effects of the forces represented by AB, AC; that is, three such forces would together produce equilibrium.

It is sometimes convenient for purposes of explanation to conceive of a single force as being made up of two or more forces. The process of finding two or more forces, which would produce the same effect as a single force, is known as the **resolution of forces**, and is the converse of the composition of forces. Thus, in fig. 7, a single force AR, might be resolved into, and replaced by, two forces AB, AC; or in fig. 5, a single force AR might be resolved into two forces AC, AB, in each case without producing any change in the effect produced.

In a future lesson we shall consider some practical applications of the rule known as the parallelogram of forces.

EXERCISES.

1. A body is acted on by forces of 8 and 15 lbs. in one direction, and by forces of 12 and 10 lbs. in the opposite direction; find the resultant force.
2. A ship is acted on by its engines and by a current, both carrying it towards the north-east, and also by a wind blowing directly from the north-east. The velocity produced by the engines alone would be 12 miles an hour, by the current alone 2 miles an hour, and by the wind alone 8 miles an hour. In what direction will the ship move, and with what velocity?
3. Find the resultant of two forces of 8 and 6 lbs. respectively, acting on a point at right angles to each other.
4. Two forces of 18 and 24 lbs. are to act on a point at the same time, what is the greatest resultant they can have, and what is the least? Can you arrange them so as to have a resultant of 30 lbs.?
5. Is it possible for a body to be at rest if acted on only by three forces, having magnitudes of 3, 4 and 8 lbs. respectively?
6. You have the means of using forces of 3 lbs., 7 lbs. and 8 lbs. respectively; describe how you would combine them to give each of the following resultants, 1 lb., 2 lbs., 4 lbs., 5 lbs., 12 lbs.
7. Mention several pairs of forces into which a single force of 15 lbs. might be resolved, representing each case by a diagram.
8. Draw two lines from the same point, and inclined to each other, and let them represent two forces; then draw a third line to represent a force which, with the other two forces, will keep the point in equilibrium.

LESSON 9.

THE THIRD LAW OF MOTION.

In order to stretch a piece of indiarubber it is necessary to apply force at each end; the force at one end acting in the opposite direction to that at the other. The indiarubber may be stretched by taking it in the two hands and pulling the ends in opposite directions. Or one end may be fastened to a heavy stone, and the other end pulled by the hand: the hand exerting a force in one direction, the stone exerting a force in the opposite direction. Or, thirdly, one end of the indiarubber may be fastened to a nail in the wall, and the other pulled with the hand, and so it may be stretched. The nail may be considered as exerting a force equal to that of the hand, but in an opposite direction. The force exerted by the nail, or any other body under like circumstances, is usually called its **reaction**, being opposed to the **action** of the other force. A cord or chain might have been used instead of a piece of indiarubber in the above experiments. When a force was exerted on one end of the cord or chain, an equal but opposite force would be exerted at the other end by the body to which it was attached. This would be shown by the tension or tightening of the cord, and its tendency to break under the action of the two forces. And if the hand took hold of the nail in the wall directly, and pulled on it, the nail would pull or react with an equal, but opposite, force on the hand.

Again, to flatten an indiarubber ball, it is necessary to press it on two opposite sides. This may be done by the two hands, or the ball may be laid on the table and pressed downwards by the hand. In the latter case the flattening of the ball proves that the table exerts a force in opposition to that exerted by the hand; or, in other words, there is a reaction by the table equal and opposite to the action by the hand. If the hand is pressed directly on the table, there is a similar reaction by the table, just as much as when the ball is interposed.

Newton's third law of motion states as a general rule what we have found to be true in the particular cases to which we have just referred. **To every action there is always an equal and contrary reaction.** To this short statement of the law Newton adds a few words of explanation, which, if we correctly understand them, will render more precise our ideas concerning the action of force. In the first place, we are to observe that there are always two bodies concerned in the action of a force. In our ordinary, but not strictly correct, language, we speak of one body as acting or exerting the force, and the other body as being simply acted on. But we ought rather to consider the force as a mutual action between the two bodies, that is to say, exerted equally by each body on the other. The force may be such as to tend to push the two bodies apart, as in the case of the hand pressing on the table, or to pull them towards each other, as in the case of the hand pulling on the nail. But in every case its action on the one body is always accompanied by an equal but opposite reaction on the other. The full statement of Newton's third law is as follows:—To every action there is always an equal and contrary reaction; that is to say, the actions of two bodies upon each other are always equal, and in opposite directions.

If a magnet and a piece of iron are floated upon water near each other, they will approach each other on account of the attraction between them. If the magnet is held fast, then the iron will move up to the magnet; while if the iron is held, and the magnet allowed to float freely, the magnet will move up to the iron. So that it appears that the force draws the magnet towards the iron, as well as the iron towards the magnet; or, as we have stated it above, the force acts equally on both bodies, but in opposite directions. A very interesting experiment described by Newton will serve to prove that the action and reaction are equal to each other. He floated a magnet and a piece of iron side by side, on water, but separated them from one another by means of an interposed obstacle. And he argued that if the iron were drawn towards the magnet with a greater force than the magnet were drawn towards the iron, there would not be equilibrium, but the greater force would prevail and produce motion, and the iron would push the obstacle and the magnet before it through the water. But Newton found, as anyone else will find who tries the experiment, that there is perfect equilibrium; and he was therefore justified in asserting that the force acted equally in both directions, or that action and reaction are equal and opposite. As we have pointed out, it is more correct to consider that a single force is acting on both bodies, than that there are two distinct forces acting. And action and reaction are rather to be regarded as two aspects of the same process from different points of view, than as two distinct processes. This feature in the character of a force has been well compared to a trading operation, in which one person buys what another sells. There is only one transaction, but in it two persons are concerned; the one person regarding the transaction as one of buying, the other from his point of view regarding it as selling.

If the two bodies on which the force acts have equal masses, and are both equally free to move, they move towards each other with equal velocities. If the masses are unequal, and the bodies both free to move, the one having the smaller mass will move with the greater velocity. This follows from what we have explained in a previous lesson, that if equal forces act on unequal masses the velocities produced are unequal, but the momentum produced in the one case would be exactly equal to that produced in the other. A man sitting in a boat, and pulling a rope attached to another boat, similarly placed and equally loaded, would cause both boats to move towards each other with equal velocities, and they would meet midway. But if his own boat were more heavily loaded than the other boat, the latter would move the faster, and travel over the greater distance. When a cannon is fired, the gas produced by the combustion of the powder presses with equal force on the cannon and on the ball. Both are set in motion, the ball with its comparatively small mass being started with a great velocity forwards, the cannon with its much larger mass being driven backwards with a proportionally smaller velocity. This backward motion of a cannon is known as the "recoil," and in the case of a gun, as the "kick." The momenta of the two bodies are, however, precisely equal; for if the mass of the cannon be 250 times as great as that of the ball, the velocity with which the ball is started will be 250 times as great as that with which the cannon recoils.

The tension of the strap attaching a horse to a carriage shows that there is a reaction as well as an action; that the carriage pulls on the horse just as much as the horse does on the carriage. As a rule, the carriage is much more easily moved than the horse is, and, therefore, the carriage moves towards the horse very much faster than the horse moves towards the carriage. But when the road is very rough, so that there is a great resistance to the movement of the carriage, —or, on the other hand, when the road is very slippery, and the horse cannot obtain a good foot-hold,—the equality of the action and reaction is well illustrated; for in either of those cases, in spite of the great tension of the strap, there is no motion produced, or, very possibly, the horse may slip and be pulled towards the carriage.

It is very easy to find abundant examples, in addition to those given above, to illustrate the action of this law. It will explain, for instance, why a person in a boat or carriage pushing against the front cannot set the boat or carriage in motion. Some further examples are referred to in the following exercises.

EXERCISES.

1. A person jumping off a stool pushes the stool backwards at the moment of jumping. Explain this.
2. Why does a person wishing to push anything forward with great force plant his feet firmly on the ground, or set his back against a wall?
3. What effect on the motion of a ship would it have if a gun were fired (*a*) from the stern, (*b*) from the bow?
4. A man sitting on the pan of a balance to be weighed pulls on the chains by which the pan is suspended; will that make any difference in the weight indicated?
5. Why is it difficult to push along a heavy body when standing on ice?
6. Explain why it is that a person standing in a box cannot raise himself up by lifting the sides. What will happen if he pulls with sufficient force?
7. Explain on the principle of the third law of motion how a ship is moved by means of paddle-wheels.
8. If an indiarubber ball, filled with air until it is very much stretched, is laid on a smooth table, and the air allowed to escape through a small hole, the ball is moved backwards. Explain this, and explain on the same principle how a rocket is lifted.

LESSON 10.

THE LAWS OF MOTION.

In the preceding lessons we have considered the three laws of motion as they are set out by Newton, and will now supplement what has been there stated by some general ideas in connection with those laws. In the first place, let us endeavour to understand clearly what is meant by the term **law** as used in science. We speak of natural laws, or laws of nature, to distinguish them from what we may call artificial or arbitrary laws, such as the particular laws of a nation or country. The latter are rules or regulations imposed upon the nation by some authority, such as an assembly or parliament.

These laws may be altered from time to time, so that at one period a particular action may be quite contrary to the law, whereas at another period it might be perfectly lawful. Now the laws of nature are not of this kind; they are not rules framed by any man or number of men which certain bodies must obey. A man may discover a law of nature, but cannot make one. A law of nature is simply a statement that under certain circumstances certain events will take place. Such a statement is based solely on the result of more or less experience, and its truth will have to be proved or disproved by an appeal to experience. For example, it is a matter of common experience that bodies which are unsupported fall to the earth. The truth of such a statement is proved not by any amount of reasoning, but by a simple appeal to our experience. Again, it is a law of nature that friction between two bodies produces heat. This is a statement which embodies the results of a large number of observations, and we can satisfy ourselves that it is true by observing for ourselves the effects produced by the action of friction. "Bodies expand when heated" is a statement of what has been observed to take place in a countless number of cases where the temperature of bodies has been raised.

All our experience of nature shows us that events happen regularly and uniformly: that is to say, the same causes always produce the same results. So that from our experience and knowledge of the laws of nature, we can, in many cases, predict what will be the result produced by the action of certain causes. We know, for example, that oil will float on water, that if we put our fingers in the fire they will be burnt, that a sounding bell is in a state of vibration, that a certain mixture of air and coal-gas will explode when a light is applied, &c.; and the knowledge of these facts is the result of experience—either the experience which we have ourselves gained by actual observation, or the similar experience of others, the results of which we have learned from books or other sources. And the value of a knowledge of the important laws of nature is, that it teaches us how we must arrange the causes in order to produce certain effects. We learn, for example, that in order to make a pendulum go more quickly we must shorten it, and that in order to bend a piece of glass we must soften it. And it follows that the more complete and perfect is our knowledge of the laws of nature, the more able we shall be to produce any result which we may desire. It is in this sense that Lord Bacon's famous saying—"knowledge is power"—is true. We may say, therefore, that a law of nature is nothing more than a general statement of the observed order of events, and is based on universal experience.

Our knowledge of the facts of nature is derived either from simple **observation** or from **experiment**. In the one case we obtain our knowledge by taking advantage of any opportunities which may occur to observe that certain results follow the action of certain causes. In the other case—that of experiment—we make the opportunities, arranging the circumstances to suit our convenience, and to exhibit most clearly the effects which we wish to study. To satisfy ourselves, for example, that bodies expand when their temperature is raised, we might examine carefully the condition of a stone during the day when exposed to the direct rays of the sun, and its condition at night; or we might measure the length of the bars of a fire-grate when cold, and again when heated by a fire. But it would be much more convenient to arrange an apparatus in which we could lay prepared bars of different substances, and then heat them by a lamp, the expansion being shown by the movement of an index bar. In the latter case we should be said to have arranged and performed an experiment; in other words, we should have made our observation under favourable circumstances arranged by ourselves.

The laws of motion are statements as to certain relations existing between the condition of bodies and the forces acting on them. The truth of these statements cannot be directly demonstrated, but the fact that none of them have been disproved is something to be said in their favour. The most satisfactory testimony to their truth, perhaps, is afforded by the fact that calculations as to the motions of bodies, made on the assumption of the truth of these laws, are in all cases verified by experience, if all the causes have been taken into consideration. An astronomer, for instance, calculates the particular hour and minute when the earth will pass between the sun and moon so as to eclipse the latter; and he also traces the paths of the sun, earth, and moon so carefully, as to be able to say what portion, and how much, of the moon's surface will be obscured. He proceeds with his calculations on the assumption that these bodies will all move uniformly in straight lines, except in so far as they are compelled to deviate from their directions by the action of external forces. He ascertains what forces will act on each body, and traces out the change of motion which will be produced by each. And, thirdly, he bears in mind that attraction between any two of these bodies acts equally on each; that with every action he must allow for a contrary and equal reaction. And the fact that the predictions of astronomers as to the occurrence of eclipses, the return of comets, &c., have come true time after time, furnishes the most valuable testimony to the truth of these laws of motion, as well as to the advantages gained by a knowledge of them.

The first law of motion states that the condition of a body will remain unchanged in the absence of all external force. It furnishes us, in fact, with a definition of force, viz., any cause which can alter a body's state of rest or motion. The second law asserts that the effect produced by the action of a force is the same under all circumstances, and depends only upon the magnitude and direction of the acting force. And it therefore provides us with the means of measuring forces; since we have only to measure the effects produced by various forces, and compare these effects one with another—the forces themselves being proportional to the effects which they produce. The third law of motion teaches us to regard every force as a mutual action between two bodies, its action on each body being equal in amount, but opposite in direction, to that exerted on the other. And whatever the nature of the bodies on which the force acts, or whatever the variety of force concerned in the action, experience shows that these three laws of motion are true in every case.

EXERCISES.

1. A bird rises by pressing its wings with considerable force against the air under them. What law of motion does this illustrate?
2. State any "law" that you know, which has been observed with regard to the action of capillary force.
3. If a gun is held horizontally in front of a wall whilst being fired, will the bullet strike a point on the wall at the same horizontal level as that of the gun? State the law of motion on which your answer is based.
4. A bullet fired from a rifle at a short distance perpendicularly into a quantity of water, is found to be more or less flattened out into a sheet. What does this illustrate?
5. A boatman rowing up a stream, finds an advantage in keeping near the shore. Can you give any reason for this?
6. Why does a heavy top spin longer than a light one?

LESSON 11.

THE FORCE OF GRAVITY

If any body is raised from the ground, and then left to itself without support, it will fall down again to the earth. The apparent exceptions to this rule—smoke, balloons, &c.—are not in reality exceptions, for such bodies fall in a vacuum as readily as any other body. The reason why they do not fall in air is because of the greater density of the air, and therefore its greater pressure towards the earth. In a pushing crowd of persons, the stronger ones will gradually make their way towards the front, while the weaker ones will be gradually forced backwards. In the same manner, the air pushing with greater force than the smoke or balloon, gradually forces these latter upwards away from the earth. We may, therefore, justly say that all bodies in the neighbourhood of the earth's surface have a tendency to fall to the earth. This tendency is accounted for by assuming that the earth exerts a force of attraction on bodies, and to this force the name of **gravity** is given. To the effect produced by the action of gravity, viz., the tendency of bodies to move towards the earth (and, as we shall afterwards explain, towards each other also), the name of **gravitation** is given. This tendency is shown in the case of a supported body by a certain pressure on its support, which pressure is directed towards the earth, and is known as the **weight** of the body. At every part of the earth's surface visited by man, and at the greatest height in the atmosphere to which he has ascended, gravity is found to act, producing either motion or pressure.

The motion of bodies allowed to fall freely at any place, and the direction in which the pressure of supported bodies acts, is found to be always vertically downwards, or, in other words, directed towards the centre of the earth.

That the action of gravity is not confined to the surface of the earth is proved by the fact that in balloons, as well as in the deepest mines, bodies have weight, and fall downwards when unsupported. The force of gravity, in fact, appears to act, not from the surface, but from the centre of the earth. So far as our experience goes, gravity is universal, since all bodies we know have weight, and tend to fall. A string supporting a weight at its extremity hangs in a vertical line, which, if produced downwards, would pass approximately through the centre of the earth. Such a line is perpendicular to the horizontal surface of a liquid at rest, and is the direction in which the force of gravity acts on the weight. If the string supporting the weight breaks, the body falls in the direction of the straight line in which the string hung.

But we have so far considered only one side of the matter, viz., the action of the earth on a body. According to the third law of motion, there must be an equal reaction exerted by the body upon the earth, and tending to produce a motion of the earth in the opposite direction to that of the body. Gravity must in fact be a force exerted mutually between the earth and every body on or near the earth. But it may be asked why, if a fallen stone attracts the earth with as much force as the earth attracts the stone, the earth does not rise to meet the stone. The earth really does move towards the stone, and the momentum of the motion of the earth towards the stone is exactly equal to the momentum of the motion of the stone towards the earth. The earth's mass, however, is so enormously greater than that of any stone, that the momentum in each case will not be the same, unless the velocity of the stone is enormously greater than that of the earth. The distance through which the stone will move towards the earth will, therefore, also be enormously greater than the distance through which the earth moves towards the stone. For example, suppose the mass of the earth to be 100,000,000 times as great as that of a body falling towards it. Then the velocity of the earth (and the distance through which it moves) would only be the $\frac{1}{100,000,000}$th part of the velocity of the body. When it is stated that such a body would weigh millions of millions of tons, we can understand how inconceivably small must be the movement of the earth, caused by the reaction to the falling of an ordinary stone.

Before the time of Sir Isaac Newton there was a very general knowledge of the effects of gravitation. These effects, however, were generally considered to be confined to the neighbourhood of the earth; though some philosophers had gone so far as to express their belief that it might be gravity which retained the moon in its orbit round the earth, and the various planets in their orbits round the sun. But it was left for Newton to demonstrate that gravity was indeed a universal force, acting on every particle of matter in the universe, and acting over distances so immense that they are beyond even the powers of our imagination. He discovered the law of gravitation, according to which the intensity of the force of gravity varies on different masses, and at different distances. And he was enabled to explain, according to this law, the motions of the earth and moon, the motions of the other planets and their moons, and in fact the motions of all the bodies forming what we call the solar system, at least so far as they were then known. He explained the formation of tides in the ocean covering the earth's surface as due to the attraction of the moon, and showed that a similar, but smaller, tide was produced by the force of gravity acting between the earth and sun. Since Newton's time a most striking confirmation of the truth of his assertion, that gravity acted on all bodies in the universe, has been afforded by the discovery of many examples of so-called "double stars." These consist of two stars evidently connected together in some manner, and revolving round each other, in consequence of the action of some attractive force, which we have no reason to believe to be any other than gravity.

We have pointed out, in a previous lesson, that a body in motion has a constant tendency to continue its motion in a straight line. A stone fastened to a string, and whirled round the hand, keeps the string constantly stretched by reason of this tendency—centrifugal force, as it is often called ; but for the tension of the string constantly pulling the stone towards the hand, it would not continue to move in its circular path. The moon, in its revolution round the earth, must also be kept in its circular course by some force directed towards the earth ; and this force, according to Newton, is precisely the same force as that which causes a stone to fall to the ground, viz., gravity. In fig. 8 let the large circle represent the orbit of the moon, having the earth, E, in its centre. The moon, M, has a tendency to move in the direction of the straight line M I, and, therefore, to move farther and farther away from the earth. But the effect of the force of gravity, represented by M G, must be combined with the effect of the inertia M I, and, by the parallelogram of velocities, the actual path of the moon will be from M to O. Knowing the mass of the moon, and the velocity with which it moves in its orbit, it is possible to calculate how great must be the intensity of gravity to keep it in its course. This is exactly what Newton did in order to demonstrate the truth of his law of gravitation, which we shall explain in the next lesson. The moon, in fact, may be regarded as a falling body, having a tendency to fall towards the earth, but prevented from doing so by its "centrifugal force." The same figure

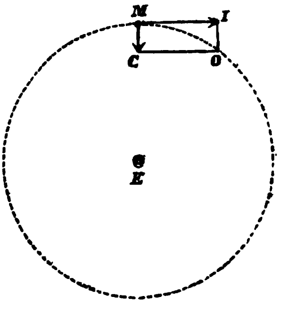

Fig. 8.

(fig. 8) would serve to explain the motion of the earth, or any other planet, in a more or less circular path about the sun as a centre. The force acting in each case towards the sun, and changing the course of the planet from a straight line, is the force of gravity.

EXERCISES.

1. Cohesion and gravity are sometimes regarded as the same force acting under somewhat different circumstances. Point out in what respects they resemble each other, and in what they differ.
2. A piece of wood is pushed down to some depth below the surface of a lake, and then released. Explain why the wood rises instead of falls.
3. Mention any facts you know to prove that gases are acted on by gravity.
4. Give some examples of gravity producing and destroying motion respectively.
5. Describe some simple experiments to prove that bodies supported so as to be unable to fall are still acted on by gravity.
6. How would you proceed to test a wall or other structure to ascertain if it were vertical?

LESSON 12.

THE LAW OF GRAVITATION.

Although the force of gravity acts everywhere, and on all bodies, its magnitude or intensity is not the same in all cases. The law according to which this variation takes place was discovered and proved by Newton. He proved that the intensity of the mutual gravitation (or tendency to approach each other) of two bodies depends upon two things, viz.—the masses of the two bodies, and their distance from each other. The greater the masses of the bodies, the stronger is their mutual gravitation; the greater the distance by which they are separated, the weaker is their tendency to move towards each other.

Gravitation is a property, not of heavy bodies merely, but of every body; and it is not a property of a body as a whole, but rather a property of each particle entering into the composition of the body. Every particle of matter in the universe attracts every other particle. Every particle of matter composing the earth attracts every particle of the moon or of a falling stone. What we usually regard as a single force acting on the moon or the stone, is the resultant of all these numberless small attractions, acting apparently on some one point, situated in a more or less central position in the body. Newton proved that for bodies spherical in shape, the resultant of these minute forces was a force equal to their sum, and acting from the centre of the sphere. The shape of the earth and of the moon, as well as of the sun and the planets generally, is so nearly that of a sphere, that this rule may be considered as practically true for all such bodies. The total action of all the particles composing the earth upon the moon, or a stone, is the same as if all these particles were collected together in one point at the centre of the earth. This will explain why bodies falling freely move in the direction of the centre of the earth, and why a string supporting a weight is pulled in such a direction. And in measuring the distances between two bodies, in order to ascertain over what distance gravity has to act, it is the distance between the centres of the bodies which must be measured. Thus the distance over which the resultant action of the earth is exerted on a stone near its surface, is about 4,000 miles: that is, equal to the radius of the earth. There is in every body a point at which the resultant of the attractive forces, acting on the particles of the body, may be considered as acting; and this point is called the **centre of gravity** of the body. This point has several important properties, to the consideration of some of which we shall devote the next lesson.

It will readily be understood that as gravity acts upon the ultimate particles of which bodies are composed, the greater the quantity of matter in a body, the greater will be the resultant effect of gravity upon the body. This is proved by the simple fact that the weight of a body depends upon its mass; that if, for example, a body be weighed, and then a portion of it cut away, the weight of the remaining portion will be less in exactly the same proportion as the mass is less than before. The mutual gravitation of two bodies having masses of 1 and 2 respectively, would be 2 (that is, 1 × 2) as compared with the gravitation of two other bodies of masses 2 and 4, the gravitation of which would be represented by 8 (that is, 2 × 4), or four times as great as in the former case, provided, of course, that the distance was the same in each case.

In all cases of attraction—magnetic, electrical, &c.—the intensity decreases as the distance between the bodies is increased. And just in the same way the heat from a fire, or the light from a lamp, grows weaker as we recede from its source. The same is the case with gravity. If this be so, the weights of bodies, and the rates at which they fall, ought to be less as we go farther from the earth's surface; and there can be no doubt that such is the case. But the difference is so small at the different places to which we have access, that we cannot perceive it by ordinary means. We have explained that the force of gravity, acting between the earth and a stone on the earth's surface, must be considered as acting from the centre of the earth, a distance of about 4,000 miles. Compared with this distance the height of a room, of a house, or of a hill is so exceedingly small, that the weakening of the gravitation will be also exceedingly small. According to Newton's law of gravitation, if a body were removed to a distance of 4,000 miles above the earth's surface (that is, 8,000 miles from the centre of the earth) the force of gravity on it (and therefore its weight) would only be one quarter of what it was at the earth's surface. If the distance separating the body from the centre of the earth were increased to three times what it originally was, the effect of gravity on it would be only one-ninth part of its former amount. It will be observed that gravitation does not decrease in the same proportion as the distance is increased, but in the proportion of the square of the distance. At six times the distance, gravitation is reduced to $\frac{1}{36}$th part of its former amount; at $\frac{1}{4}$ of the distance, to 16 times its former amount. The law of gravitation may now be stated generally as follows:—**Every particle of matter in the universe attracts every other particle with a force proportional to the product of their masses, and varying inversely as the square of their distance from each other.**

Newton sought to prove the truth of this law by showing that, if gravitation varied as stated therein, the motion of the moon in its orbit could be explained. The moon is about 240,000 miles from the centre of the earth, that is to say, is 60 times as far away from the earth's centre as a body on the surface of the earth is. Gravitation at the distance of the moon should therefore be only the $\frac{1}{3600}$th part (that is, 60 × 60 times less) of its amount at the earth's surface. Newton showed that if gravity had this intensity at the distance of the moon, it would cause the latter to move out of a straight line, and towards the earth, about 15 feet in a minute, which is the amount required to account for the moon's motion round the earth.

By means of this law of gravitation, it is possible for an astronomer to calculate by how much a comet will be deflected from its course by the attraction of a planet. Knowing the mass of the sun and the distance which a body on its surface will be from the centre, it is possible to calculate the weight which a body would have on the sun, compared with what it has on the earth. The weight of a body on the sun would, we are told, be 28 times as great as on the earth, while on the moon it would only be one-sixth as great as on the earth. This law also explains why the tide raised by the moon is more than twice as great as that raised by the sun, the smaller distance between the earth and moon more than making up for the enormously greater mass of the sun.

Since the time of Newton it has been several times proved by direct experiment that all bodies have a tendency to gravitate towards each other. And it is only because the mass of the earth is so enormously great compared with the masses of bodies on its surface, and therefore its attraction so much greater than theirs, which prevents us from more readily observing this general property of bodies to gravitate towards one another. It has been recently shown that when a large mass of lead is brought close under a small body hung from a very delicate balance, the body weighs slightly more than when the lead is removed; thus proving an attraction between the body and the lead. It has also been proved that a plumb-line does not hang in an exactly vertical line near a mountain, but that the weight at the end of the string is drawn slightly towards the mountain, by the mutual attraction between them.

EXERCISES.

1. Find the weight of a mass of iron at a distance of 4,000 miles above the surface of the earth, which on the surface weighs one hundred-weight.

2. Why should a body on the surface of the moon weigh less than the same body on the surface of the earth? What would be the weight on the moon's surface of a body which on the surface of the earth weighed 3 cwt.?
3. If the tide raised by the moon's action alone is 2½ times as great as that raised by the sun alone, compare the weight of the tide raised by the two bodies acting in conjunction with each other (spring tides) with that of the tide raised by them when acting against each other (neap tides).
4. A pendulum was allowed to swing for a certain time at the top of a mine, and then for the same length of time at the bottom, and the number of swings in each case were counted. How could it be ascertained from these experiments whether the intensity of gravity was different in the two places?
5. Why does a piece of lead weigh more than a piece of wood the same size? If both bodies were taken to a place where the intensity of gravity was less than in England, which would lose the more in weight?

LESSON 13.

CENTRE OF GRAVITY.

We have already explained the centre of gravity of a body to be that point at which the resultant of the several forces, due to the action of gravity on the particles of the body, may be supposed to act. In the case of a thin rod of uniform thickness and density, the middle point of the rod will obviously be the centre of gravity. In experimenting with such a rod—a wooden ruler or lead pencil, for example—it will be found that the rod will balance on the finger if the latter be placed under the middle point of the rod, but that it will not balance in any other position. If a string is tied round the middle part of the rod and held up, the rod will hang balanced horizontally. But if the string is attached to any other part of the rod, the latter will not rest horizontally but will incline more or less until the middle point or centre of gravity is vertically below the string. And if any body whatever be suspended by a string and allowed to hang freely from the point of suspension, it will be found that there is one position, and one only, in which the body will hang steadily, and that if it be displaced from this position it will return to it again. And if the position of the centre of gravity of the body is known, it will be seen when the body is at rest to be vertically under the string; or, in other words, a straight line drawn in continuation of the direction of the string would pass through the centre of gravity of the body. It appears, therefore, that in order to prevent a body from falling, its centre of gravity must be supported either from above or below.

A simple practical method of determining the position of the centre of gravity of a body is founded on the principle just stated. For example, suppose it is required to find the centre of gravity of an irregularly shaped piece of cardboard (fig. 9). Suspend it from

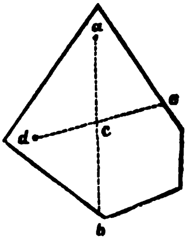

Fig. 9.

any point *a* by means of a piece of string, or by a pin through the card, so as to let it freely take its natural position. Then from the point of suspension *a* draw a vertical line *acb* on the card; the direction of which can be most readily ascertained by means of a small plumb-line, *e.g.*, a small weight hung to the end of a piece of string. This line will pass through the centre of gravity of the cardboard. Then suspend it from any other point, such as *d*, and again draw a vertical line *dce* from the point of suspension, which line will also pass through the centre of gravity. The centre of gravity being in both these lines, can only be at the point *c*, where they cross. And it will be found that from whatever point the body is suspended, a vertical line drawn from the point of suspension will always pass through the point *c*. It need scarcely be pointed out that the centre of gravity will not be on the surface of the body, but mid-way between the two faces of the cardboard.

The centre of gravity of regularly shaped bodies, of uniform thickness and material, can frequently be most readily found by geometrical construction. Thus the centre of gravity of a circular disc, or a sphere, is at the centre of each; of a square or oblong, at the intersection of the diagonals.

If a hole were made through the cardboard, represented in fig. 9, at the centre of gravity *c*, and a smooth knitting needle inserted and held horizontally, it would be found that the card would remain in equilibrium on the needle in any position. But if the needle were inserted through a hole made in any other part of the card, such as *a* or *d*, the card would only remain in equilibrium in two positions, viz., when the centre of gravity was directly above, or directly below, the point of support. When the card was balanced with its centre of gravity vertically above the point of support, its equilibrium would be very unstable or unsteady; the least displacement to the one side or the other of its position of equilibrium would cause it to fall down into the more steady or stable position of equilibrium, where its centre of gravity would be vertically below the point of support. After any displacement from this latter position the body returns to the same position again when released. A body is said to be in **stable equilibrium** when, after displacement, it returns to its original position, with its centre of gravity as low as possible. A body is said to be in **unstable equilibrium** when, after a slight displacement, it does not recover its position, but moves into some more stable position, with its centre of gravity lower than before. A body is said to be in **neutral equilibrium** when, like the cardboard with the needle through its centre of gravity, it remains in equilibrium in any position.

The centre of gravity of a body always tends to get to its lowest position; and a body will be in stable or unstable equilibrium according as its centre of gravity is raised or lowered by a slight displacement of the body. In the former case, when the centre of gravity is raised, the body when released will fall back into its original position; in the latter case, where the centre of gravity is lowered, it will not return. In fig. 10, the cone represented at A is in stable equilibrium, the dotted lines showing the path of the centre of gravity *c* when the cone is tilted to either side. The cone at B is, on the other hand, in unstable equilibrium; whilst a ball on a level surface would represent a body in neutral equilibrium. The cone at C, where the vertical line drawn from the centre of gravity *c* falls within the area of the base, will fall back into the position represented by A; whilst the cone at D, where the vertical line drawn from the centre of gravity falls outside the base, would fall on to its side.

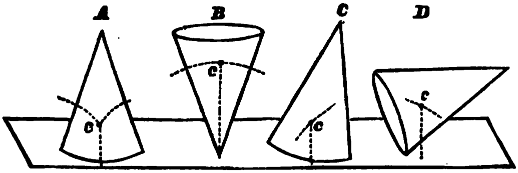

Fig. 10.

We may briefly sum up this short account of the properties of the centre of gravity of bodies, by saying that in order that a body may remain in equilibrium its centre of gravity must be supported; or, in other words, a vertical line drawn from the centre of gravity must pass through some part of the portion of the body which is supported. In the second place, in order that the body may stand firmly, its base should be as large, and its centre of gravity as low, as possible; and the body should be so arranged that any force tending to displace it from its position of equilibrium would have to raise its centre of gravity.

EXERCISES.

1. Show how to find the centre of gravity of an ellipse, and of a triangle by geometrical construction.
2. Why does a person stand more steadily on two feet than on one foot?
3. A person carrying a heavy load on his back leans forward, whilst a person carrying a weight in one hand leans towards the other side. Explain these facts.
4. Why should heavy goods be packed at the bottom part of carts and of vessels?
5. In what kind of equilibrium is a cylinder when standing on end, and when lying down?
6. Explain why a small boat is so much more easily upset when persons are standing up in it than when they are sitting or lying down.
7. Describe a simple means of finding the centre of gravity of a poker or shovel.

LESSON 14.

WEIGHT.

Weight is the name given to the tendency which every body in the neighbourhood of the earth has to move downwards towards the earth's centre. This tendency is shown by the pressure which a body exerts on its supports; that is to say, on other bodies which resist its motion. This tendency is due to the action of gravity, and its strength varies as the intensity of that force. The weight of the body also depends upon the existence of matter; in the absence of matter there could be no weight. We know by experience that bodies differ considerably in weight, and the cause of this difference we have already to some extent pointed out. The weight of a body depends upon its mass, or the quantity of matter which it contains. This is very evident so long as we deal with the same kind of matter. One pint of pure water has exactly the same weight as another pint of pure water, provided both are weighed under the same circumstances. And a quart of pure water has double the weight of a pint of water; while a cubic foot of lead has 144 times the weight of a cubic inch of lead. Newton proved, by careful experiments with pendulums, that equal masses of different kinds of matter, under the same circumstances, are equally acted on by gravity, and therefore have the same weight. It follows that in the case of unequal masses the weights will be in exactly the same proportion as the masses, without respect to the kind of matter composing the masses. Thus if one piece of lead just balances three equal pieces of wood, the mass of the lead, as well as its weight, is equal to that of the three pieces of wood together, and is three times as great as that of one of the pieces.

But whereas the mass of a body would be the same in every place and under all circumstances, the weight of the body would necessarily be altered if the intensity of the force of gravity changed. The weight, or downward tendency, of a body would be greater if the force of gravity acting on it became stronger, and less in the case of gravity becoming weaker. The weight of a body is therefore not exactly the same at two places at different heights, but decreases as the body is carried higher. The variation for all ordinary heights is, however, so exceedingly small that we may practically neglect it. On account of the deviation of the shape of the earth from that of a sphere, a body at the equator is 13 miles farther from the centre of the earth than it would be at the poles. A body carried from the equator towards one of the poles is, as it were, all the time going down a hill and getting nearer to the centre of the earth, and its weight is gradually, though very slightly, increasing. But there is another cause which also helps to make the weight of a body least at the equator and greater at the poles, viz., the rotation of the earth. All bodies on the earth's surface are being whirled round with the earth at an enormous rate, and therefore manifest "centrifugal force," or a tendency to move away from the earth, and to continue their motion in a straight line. This tendency acts in opposition to the gravitation of a body towards the earth, and therefore lessens the weight of a body. And since the centrifugal tendency of a body increases as the velocity of rotation increases, it will act most strongly on a body at the equator where the earth is rotating fastest, and diminish on passing from the equator towards either pole. Thus the weight of a body at the equator is less than at the poles, partly because in the former place the body is farther from the centre of gravity of the earth, and still more because of its centrifugal tendency. From pendulum experiments, made in various places on the surface of the earth, it is calculated that the variation in the weights of bodies is such, that a body weighing 194 lbs. at the equator would, if carried to the poles, weigh 195 lbs.

The weight of a body is distinguished from the mass of a body by its possibility of change. The weight of a body may change; the condition of the body—solid, liquid or gaseous—may change; so also may its size, shape, colour, hardness, &c.; but the mass of the body is unalterable, and is in fact the only unalterable property of matter.

The standard of weight for this country is the imperial standard pound, defined by Parliament to be a certain platinum weight, of which our ordinary weights are supposed to be accurate copies, so far at least as their weight is concerned. This piece of platinum is what we have before referred to as our standard of mass; and, correctly speaking, it is in reality a standard of mass (or quantity of matter), rather than of weight (or downward tendency).

The instrument generally employed for weighing bodies is known as a pair of scales, or a balance. The body to be weighed is put into one pan, and then standard weights are placed in the other pan, until the pans hang equally balanced. The weight of the body is then said to be equal to that of the weights in the other pan, assuming, of course, that the balance itself is perfectly accurate. A little consideration will show that in this method of determining the weight of a body, no account need be taken, even in the most exact experiments, of the locality in which the process of weighing is performed. If the masses in the two pans balance each other in one locality, they would do so just the same in every other locality, however different the force of gravity might be. This being the case, it is obvious that the ordinary weighing scales do not really determine the weights or downward tendencies of bodies, but only serve to determine equal masses. That is to say, in the process of weighing sugar, for example, the scales will show when the mass of the sugar in the one pan is equal to the mass of the weights in the other. But when the weight of a body is determined by noting the amount of elongation of an elastic spring on which it is hung, the case is different. Suppose a body were hung on an accurately graduated spring balance and carried to different parts of the world. As the downward tendency or weight of the body varied in different places, so would the indications of the balance vary. And if the balance were originally graduated to weigh correctly in London, it would not weigh correctly at any other place having a latitude different from that of London. A spring balance, in fact, is an instrument for measuring force; a pair of scales is an instrument for measuring mass.

EXERCISES.

1. The British standard pound contains 7,000 grains, and the French standard of mass (and weight)—the kilogramme—is equal to about 15,432 grains. Express the weight of a kilogramme in British units.
2. If a merchant bought goods at Bombay and sold them at Liverpool, the weight in both cases being determined by spring balance, would he gain or lose by the difference in weight at the two places?
3. The elasticity of a spring varies somewhat with the temperature; will that affect the accuracy of its indications?
4. Can you give any reasons for the standard pound weight being made of platinum?
5. Describe an experiment to prove that the air has weight. A cubic foot of air from the top of a mountain would weigh less than an equal volume at the bottom; why?
6. What is the difference between *specific gravity* and *weight?* If the specific gravity of platinum is 21.5, find the weight of a cubic inch of that substance; the weight of a cubic foot of water being 1,000 ounces.

LESSON 15.

MEASURES OF FORCE.

The magnitude, intensity, or amount of a force is a measurable quantity, and as such is capable of being represented diagrammatically by a straight line. In our lesson on the composition of forces we made use of straight lines to represent forces, the lengths of the lines being proportional to the magnitudes of the forces. The effect produced by a force is the proof we have of its existence and action. From the second law of motion we learn that the effect produced by a certain force (*e.g.*, change of momentum) is proportional to the magnitude of the force, and it is on this principle that the measurement of force is based. To compare the magnitudes of any two forces, therefore, it is necessary to measure the effect produced by each under similar circumstances, and then to compare these effects.

The most common method of indicating the magnitude of a force is to compare it with the force of gravity, which affords a constant and convenient standard of reference. A portion of matter raised above the earth's surface has a tendency to fall on account of the action of gravity. This tendency (or weight) is expressed in terms of the standard pound; in other words, the force which urges the body towards the ground is said to have a magnitude of so many pounds. Any force acting in the opposite direction to that in which gravity acts, and just sufficient to prevent the body from falling, would necessarily be equal in magnitude to the force of gravity acting on the body, that is, to its weight. For if the upward force were greater than the downward tendency of the body, the latter would move upwards with a certain velocity; whereas if the downward tendency were the greater, the body would fall, though not so quickly as if gravity alone were acting on it. The equilibrium of the body is the test of the equality of the two opposite forces. The magnitude of a force may thus be determined by ascertaining what weight it can support against the action of gravity. And the magnitude of a force may be expressed in the same terms as are used to express the weight of a body. A force of 100 pounds would, according to this method, mean a force which, when acting in opposition to gravity, could just support a body weighing 100 pounds.

This method of measuring forces—known as the **gravitation measure**—is that which has been commonly adopted by engineers and practical men generally. Suppose, for example, it is required to determine the tenacity of a particular variety of steel. A rod is made of the material, having a sectional area of one square inch, or some other definite amount. The upper end of the rod being securely fastened to some means of support, weights are hung on to the lower end until the rod breaks. Then the amount of tenacity (or, what comes to the same thing, the magnitude of the force of cohesion opposing the breaking) of a rod of this size and material is expressed by the weight it was just able to support without breaking. Thus it is said that the tenacity of a bar of steel one inch square varies from 50 to 60 tons. The elastic force exerted by a steel spring on a weight suspended from it is equal to the weight which it supports. If a weight of 28 lbs. hung to a spring stretches it one inch, the elastic force of the spring which prevents it being further elongated has a magnitude of 28 lbs. If such a spring were then fastened by one end to some body which it was desired to drag along the ground, and the force applied to the other end of the spring, an elongation of one inch would indicate that a force of 28 lbs. was being exerted on the body. The force employed to drag the body in that case would, if applied in another way, be sufficient to support a body weighing 28 lbs. If the amount of elongation of the spring by various weights hung from it in succession were measured, the spring might then serve as an instrument for measuring the intensity or magnitude of forces. It would also serve as a convenient means of determining the weights of bodies; for which purpose it is usual to attach a pointer to the spring, in such a way that when the spring is stretched the pointer moves over a graduated scale. Such an instrument is usually known as a spring balance; but instruments constructed on the same principle, and employed for the measurement of various forces (*e.g.*, the force employed by a horse to drag a cart along various kinds of road), are called **dynamometers**, or force measurers.

This gravitation method of measuring force, useful and convenient as it is, is open to the objection that it is not absolutely exact. The weight of a certain mass of matter varies in different latitudes, owing to the variation in the intensity of gravity. The force which will just support a body having a certain mass in one latitude, would not be exactly equal to the force which would just support the same body at a place having a different latitude. An amount of force which at the equator would be sufficient to support a mass weighing (in England) 195 lbs., would at the poles only just be able to support a body weighing 194 of such pounds. If a standard pound weight were hung to a delicate spring and carefully carried about to different regions of the earth, the amount of elongation of the spring, and therefore the force exerted by the spring on the weight, would vary. And it will be seen at once that it must be incorrect to speak of the force exerted by the spring on the weight in every place as a force of one pound. The magnitude of a force expressed in gravitation units (pounds, &c.) is only exact when the locality in which the measurement was made is stated, and the intensity of the force of gravity at that place taken into account.

There is another method of measuring the magnitude of a force which is quite independent of gravity, and in fact of every other force, and it is therefore called the **absolute measure of force.** It is perhaps not so convenient for ordinary purposes as the gravitation measure, since it is not so easily carried out. But where strictly scientific accuracy is required in the measurement of a force, the absolute method is the one employed. It is based directly on the principle of the second law of motion, viz., that **change of motion of a body is proportional to the magnitude of the acting force.** If two forces act in succession on a body for equal intervals of time, and produce different velocities in the body, the forces have different magnitudes; the one which produces the greater velocity having the greater magnitude. Suppose, for example, that a force acting on a body for one second caused the body to be moving at the end of the second with a velocity of 20 feet per second; while a second force acting for a second on the same body produced a velocity only of 5 feet per second; the former force would have a magnitude four times as great as that of the latter. It has been found by experiment that the force of gravity at the equator, acting on a raised body for a second, can set it in motion with a velocity of 32.1 feet per second, while at Edinburgh the velocity at the end of a second would be 32.2 feet. The intensity of gravity at the equator, compared with its intensity at Edinburgh, is as 321 to 322.

EXERCISES.

1. How would you ascertain the relation between the intensity of cohesion in cotton and in silk?
2. A block of wood rests on a table, and a string passes from the block over a small pulley-wheel at the edge of the table, and hangs down with a hook at its end. How would it be possible by means of this arrangement to measure the amount of friction between the block and the table?

3. A magnet at a distance of one foot from a piece of iron is found to produce on it, in one second, a velocity of three feet per second, while, when the magnet is two feet distant from the iron, the velocity produced is only nine inches per second. Compare the force of the magnet in the two positions.
4. The force of gravity at the earth's surface produces in a falling body a velocity of 32 feet per second; the force of gravity acting at the distance of the moon would produce a velocity of about 30 feet per minute. Compare the intensity of gravity at the two places.

LESSON 16.

FALLING BODIES.

It may readily be observed that a body falling freely moves faster and faster as it approaches the ground. The velocity of the body is constantly increasing, and its amount depends upon the time during which the body has been falling. Velocity which is constantly increasing is often called accelerated velocity, to distinguish it from uniform velocity. This constant acceleration is seen not only in the case of a body falling freely through the air, but also in the case of bodies rolling down inclined surfaces, as, for example, a stone rolling down a hill. On the other hand, a body thrown upwards has its velocity constantly retarded, moving more and more slowly until at last its upward motion is altogether stopped. In the case of a falling body there is, therefore, a constant addition to its velocity, whilst in the case of a rising body there is an equally constant subtraction from its velocity. If gravity is a force acting constantly and continuously on bodies, this variation in the velocity of a body moving upwards or downwards is just what might be expected from a consideration of the second law of motion.

The difference between the intensity of gravity at the earth's surface, and its intensity at the greatest height with which we have practically to deal, is so exceedingly small that we may neglect it altogether, and consider the intensity as uniform. Let us first consider the case of a body released from a state of rest at some distance above the earth's surface. It begins to move downwards towards the earth, and by the end of the first second from the time of starting it will be moving with a certain velocity, which has been found by experiment to be about 32 feet per second. If gravity then ceased to act, the body would continue uniformly on its downward course with this velocity; but as gravity still continues to act on the body, the velocity produced during the second second will be added on to that produced during the first. By the second law of motion the effect of a force on a body does not depend at all upon the previous state of the body; so that if gravity was able in one second to produce a velocity of 32 feet per second in a body previously at rest, it will be able to produce an equal velocity in one second in a body already in motion. At the end of the second second, therefore, the body will be moving with a velocity of 64 feet per second, of which velocity half was generated in the first second, and the other half in the second second, of the fall. And in the same way an additional velocity of 32 feet per second will be generated in each succeeding second during which the body is falling, so that at the end of three seconds from the time of starting it will have a velocity of 96 (that is, 32 × 3) feet per second, at the end of four seconds 128 (viz., 32 × 4) feet, and so on. To find the velocity of a falling body, it is only necessary to multiply 32 by the number of seconds during which it has been falling, and the product will give the velocity in feet per second. The effect of gravity on a body near the earth's surface, and free to move, may therefore be said to produce an acceleration of 32 feet per second. It would be difficult to measure the actual distance through which a body falls during several seconds, but experiments have been made which prove that the rule we have reasoned out above is correct.

In the case of a body projected upwards, the effect of gravity is opposed to the upward motion of the body in such a manner as to subtract 32 feet per second from its velocity. Thus a body projected upwards with a velocity of 100 feet per second, would, after it had been moving for one second, have a velocity of 68 feet per second (viz., $100-32$), at the end of the next second 36 feet per second (viz., $68-32$), at the end of the third second 4 feet, while during the fourth second it would come to rest and commence to fall downwards.

The space through which a body has fallen must not be confounded with the velocity which it has acquired. Thus, in the first second of a body's fall, during which it is acquiring a velocity of 32 feet per second, the space through which it moves is found to be 16 feet. It is obvious that the space traversed must be much less than the velocity at the end of the second, since the velocity, which at first was nothing, gradually increased during the second to 32 feet. If gravity ceased to act on the body after the first second, the velocity it has acquired would carry it in the next second through a space of 32 feet. But gravity acts just as strongly as in the previous second, and thus would move the body through 16 feet independently of its former motion. The actual space traversed by the body during the second second will therefore be $32+16=48$ feet. Similarly in the third second it will move over 64 feet (on account of the velocity acquired at the end of the preceding second) + 16 feet (due to the action of gravity during this second), making a total of 80 feet. Thus the space traversed will be as follows :—

1st second	16
2nd „	48 = 32 + 16.
3rd „	80 = 64 + 16.
4th „	112 = 96 + 16.
5th „	144 = 128 + 16.

In each case it will be seen that the space traversed during any one second may be found by adding 16 to the number expressing the velocity with which the body was moving at the end of the preceding second ; or the distance traversed in each succeeding second, commencing with the first, may be found by multiplying 16 by the odd numbers 1, 3, 5, 7, 9, &c.

It will doubtless be observed that we have said nothing above as to what particular body will be moving with a velocity of 32 feet per second after falling for one second. It is a matter of common observation that some bodies fall much faster than others—that a stone, for example, falls faster than a feather or a piece of paper. It may also be noticed that while the stone falls in a straight line, the paper or feather flutters about from side to side, and perhaps turns over several times in the course of its fall, as if something were interfering with its motion. This might suggest to us that possibly the resistance of the air may have a great deal to do with the difference between the velocity of a falling stone and a falling feather. A few simple experiments would soon show that such was the case. A sheet of paper, for instance, falls much more slowly when spread out, so as to offer a large surface to the air, than when rolled up into a ball. A piece of paper laid flat on the top of a penny falls as fast as the penny, since the resistance of the air cannot act on the paper. A feather or a piece of wool, laid in an open wooden box, falls quite as fast as the box, since it shields them from the resistance of the air. These experiments will prepare us for the statement that in a vacuum all bodies fall at the same rate: a feather falling quite as fast as a piece of lead. And the velocity acquired in one second by any body falling freely in a vacuum is 32 feet per second.

It was at one time stated that a heavy body must necessarily fall faster than a light one, since the fact of its being heavier showed that gravity acted with greater force on it than on the other body. Galileo, a famous Italian philosopher, who lived somewhat before the time of Newton, about 300 years ago, and who did a great deal for Mechanics, wisely determined to try the experiment by letting fall a heavy and light body from the same height. He dropped two bodies of unequal weights together from the top of the leaning tower of Pisa, and found that both reached the ground at practically the same instant. The gravitation of a pound of lead is certainly 16 times as great as that of an ounce of lead, but the greater force has the greater mass to move, and the velocity of the fall will be the same in both cases. Every one will see that there is no reason why 16 balls of lead, each weighing an ounce, dropped from one hand, should reach the ground any sooner than a single ball dropped by the other hand at the same time and from the same height. And there is no reason whatever why, if the 16 balls were fastened together so as to form one lump of lead weighing 1 lb., it should fall any faster than the 16 balls separately, or than the one ounce ball. The force of gravity acting on a body is always directly proportional to the mass, and the velocity produced in a given time would be the same for all bodies were there no resistance, such as that of the air, to interfere with the motion. The air interferes with heavy bodies as well as with light ones, but the greater momentum of the former enables them more readily to overcome the resistance.

We have in this lesson taken 32 feet per second as the velocity produced by gravity in one second; more correctly it is found to be in this country about 32·2 feet per second.

EXERCISES.

1. Why does a book fall faster than a single leaf of the book would fall?
2. A person jumps from a height holding a heavy weight in his hand; will he feel the pressure of the weight whilst he is falling through the air?
3. A stone is observed to take four seconds to fall from the top of a tower; what is the height of the tower?
4. The tower of Pisa is 130 feet high; how long will a stone falling from the top take to reach the ground?
5. An arrow is shot upwards with a velocity of 144 feet per second; to what height will it rise, and with what velocity will it be moving when it reaches the ground?
6. How long would a body continue to move upwards if started with a velocity of 256 feet per second, and how long would it be before it reached the ground again?
7. A cannon ball is fired horizontally from the top of a cliff 48 feet high with a velocity of 1000 feet per second; how far will the ball move before striking the water?
8. A body weighing 1 cwt. is allowed to fall from a height of 16 feet; what momentum will it have on reaching the ground?
9. One stone is let fall from the top of a high cliff, and then another is dropped from the same place a second after the first started. What distance will the stones be apart at the end of 4 seconds from the time the first started?
10. How far must a body fall to acquire a speed of 96 feet per second?

LESSON 17.

THE PENDULUM.

The fact that the intensity of gravity varies in different parts of the world has been proved by experiments with the pendulum, and the amount of variation has been most accurately measured by means of this simple instrument. It will be well for us, therefore, before leaving the subject of gravitation, to devote a short time to the consideration of the principles on which the action of the pendulum depends.

A pendulum may consist of any body, sufficiently heavy to move without any considerable retardation of its motion by the resistance of the air, attached to a fine cord or wire. A body so suspended will hang when at rest in a vertical direction, with the weight at the end of the cord—the "bob," as it is called—in the lowest possible position. Any displacement must therefore necessarily raise the bob more or less (fig. 11), and on being released it will fall back again into its original position. The bob may therefore be considered as a falling body, which, instead of falling freely in a straight line, is compelled by its connection with the cord to describe a circular path.

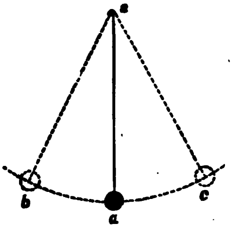

Fig. 11.

Like other falling bodies its velocity increases as it falls, and is greatest at the lowest point of its fall. By reason of inertia, the pendulum which has fallen, say from *b* (fig. 11) to *a*, and thus acquired a certain velocity, is carried forwards on the other side of the vertical line towards *c*. Were there no resistance from the air, and the friction against its point of support *s*, it would rise as high on this side as the point from which it had fallen on the other. Then falling back again towards its position of equilibrium, it will continue to oscillate backwards and forwards about that position, each time traversing a distance somewhat less than the last, until at length the resistances destroy its motion altogether, and it hangs at rest.

If the arc through which the pendulum swings is not very large, the time required for an oscillation will remain very constant, even though the length of the oscillation may diminish until it can scarcely be observed at all. For as the arc through which the pendulum swings becomes less, so also does the velocity of its motion diminish, and in the same proportion. It is on this fact that the value of the pendulum as an instrument for measuring time depends. Galileo is said to have been the first to make this important observation concerning the swinging of a pendulum, and to have been led to it by observing the oscillations of a lamp suspended from the ceiling of a church. He carried out a number of experiments with pendulums of different lengths, and having different weights at their ends; and, as the result of these experiments, he was able to show that the time of oscillation of a pendulum depends on the length, but not on the weight. If two cords of equal length are taken, and to the end of one a weight of 1 lb. is attached, while to the other is hung a weight of 2 lbs., the time of oscillation will be found to be exactly the same for each. This is indeed a consequence of the rule that all bodies fall at the same rate, for the weight at the end of a pendulum is, as we have stated, neither more nor less than a falling body, and therefore the time occupied by its fall is altogether independent of its weight. But if the cords are of unequal lengths, the periods of oscillation of the two pendulums will also be unequal; the shorter pendulum making the greater number of oscillations in a given time. A pendulum having a length of about 39 inches will swing once a second, while a pendulum having a quarter of that length (*i.e.*, nearly 10 inches), would make two oscillations per second. The material of which the pendulum is made has no influence on the time of oscillation. This was proved by Newton, who had a pendulum constructed, the bob of which consisted of a wooden box, and in this he placed in succession equal masses of different substances, with the result that the time of oscillation was the same in each case.

The movement of the pendulum depends upon gravitation, and if this varies in different places, so will the motion of the pendulum vary. An increase in the intensity of gravity would cause the pendulum to fall with greater velocity, and therefore its period of oscillation would be shortened. And it has been found by experiment that the same pendulum swings at different rates in different parts of the world. More than 200 years ago it was observed that a pendulum clock, set to measure time accurately at Paris, lost about 2½ minutes per day when carried to Cayenne in South America; and to make it keep time accurately at the latter place it was found necessary to shorten the pendulum. The law connecting the intensity of gravity with the time of oscillation of a pendulum is known; and therefore it is possible to calculate the relative intensity of gravity at two places, when the number of oscillations made by the same pendulum in a given time at the two places is known. Pendulum experiments have been made in a great many parts of the world, every precaution being taken to avoid error as far as possible. It has been found, for example, that a pendulum arranged to beat seconds at London, that is, to make 86,400 oscillations in a day, makes 86,390 oscillations at Paris; about 86,260 at a place on the equator; while at Spitzbergen, the latitude of which is about 80° N., the number of vibrations made in a day is 86,483. From these numbers the intensity of the force of gravity at the various places has been calculated, and, expressed in absolute measure according to the velocity it produces in a falling body in one second, it is as follows:—Equator 32·09 feet; Paris 32·18 feet; London 32·19 feet; Spitzbergen 32·25 feet.

EXERCISES.

1. Why is the bob of an ordinary pendulum flattened in form, and with its narrow edges in the direction in which it swings?
2. How would you regulate the pendulum of a clock which was losing time?
3. What effect have changes of temperature upon the rate of oscillation of a pendulum?
4. If a pendulum were formed by an iron ball suspended from a string, and a strong magnet laid underneath it, would that make any difference in its rate of vibration?
5. Can you give any reason for the fact that large clocks have heavy pendulums?

LESSON 18.

WORK.

Whenever a force overcomes a resistance it is said to do work. When, for example, a force is employed to raise a body from the ground—in order to accomplish which it has to overcome the resistance due to the gravitation of the body—work is done. When a person walks up a ladder or up a hill, they do work, in consequence of having to raise the weight of their body. Boring a hole in a piece of wood, and pushing a heavy box along the ground, both involve the overcoming of a certain amount of resistance due to friction, and therefore the performance of a certain amount of work. In order to bend a bow, or wind up the spring of a watch, or compress a quantity of air, the resistance offered by the elasticity of those bodies must be overcome, and, therefore, work must be done. And, in the same way, work is done in separating a piece of iron from a magnet, or a piece of paper or straw from an electrified stick of sealing-wax. It is in this very definite sense that the term **work** is used in Mechanics to mean **the overcoming of resistance**, or the production of motion against resistance. When a body is moved by the action of some force, and a resistance impeding the motion is overcome, work is said to have been done on the body. When a body in motion overcomes a resistance before being brought to rest, as when an arrow shot upwards overcomes the resistance of gravitation through a certain distance, or a bullet penetrates a target and overcomes the resistance of inertia, cohesion, and friction, the body is said to do work. In every case the force doing the work is spent in the act of doing it.

The amount of work done in any particular case depends on the magnitude of the force engaged, and the distance through which it acts. Thus more work is done in raising a weight of 28 lbs. from the floor on to a table, than is done in raising a weight of 7 lbs. to the same height; and, in fact, the amount of work done in the latter case is only one quarter as great as in the former. The reason is, that the amount of force required in the case of the 7 lbs. to overcome the resistance (gravitation) opposed to the motion of the weight, was only one-fourth of that required to raise the 28 lbs., while, in both cases, the distance through which the bodies were moved was the same. More work is done in dragging a heavy box over a certain distance along a rough road, than in moving the same box, placed on wheels, for the same distance along a smooth pavement. As before, the distance is the same in both cases; but in the one case the resistance (friction) is much greater than in the other, and the amount of force required to overcome the resistance is therefore greater. The amount of work done in any case, therefore, depends upon the magnitude of the force employed; but it depends also upon the distance through which the point of application of the force moves, in the direction in which the force acts. To raise a certain weight through one foot involves the performance of a certain amount of work; to raise the same weight two feet obviously implies double the amount of work as in the first case. And, in general, the amount of work done by a force is in proportion to the distance through which it moves the body in the direction in which it acts. The magnitude of the force employed, or of the resistance overcome, is readily known in the case of raising bodies, since it is equal to the weight of the body raised. But in other cases, when the resistance to be overcome is elasticity, friction, &c., the magnitude of the force must be determined by means of a dynamometer, or other arrangement, before the amount of work done can be estimated.

The unit of work usually adopted in this country is the **foot-pound**, and is defined as the amount of work done in raising one pound to the height of one foot against the force of gravity. A force of one pound overcoming any resistance through the distance of one foot, does one unit of work. In lifting 1 lb. to the height of 1 foot, one unit of work is done; in raising 1 lb. to the height of 6 feet, 6 units of work are done; in raising 6 lbs. to the height of 6 feet, 36 units of work are done. That is to say, to find the number of units of work done in any case, it is necessary to multiply together the number expressing the force in pounds, and that expressing the distance in feet. Sometimes, as a matter of convenience where large forces are employed, the work is expressed in foot-tons. Thus the work done by an engine in raising 250 tons of coal from a depth of 200 feet, might either be stated as 50,000 foot-tons, or as 112,000,000 foot-pounds.

Where the metric system of weights and measures is employed, the unit of work is the kilogrammetre, which is the work done in raising one kilogramme to the height of one metre.

It is often necessary to be able to measure the rate at which an engine or other agent works, and then the element of time must be taken into consideration; that is to say, we must determine how many units of work are performed in a given time. This want was most strongly felt when steam engines first came into use, in order to furnish some guide to the purchaser of an engine as to its power of doing work. James Watt, who did so much to bring the steam engine into practical use, suggested the comparison of the power of an engine with that of a horse. And in order to make this comparison with some degree of accuracy, Watt endeavoured to measure the amount of work which a horse was capable of doing, and came to the conclusion that, on the average, the power of one horse was equivalent to the performance of 33,000 units of work per minute. Later experiments would seem to show that this estimate is somewhat too high, but it is usual to consider the performance of **33,000 units of work per minute** as one **horse-power**. The power of an engine, then, is expressed as being of so many horse-power (H.P. as it is usually abbreviated.) An engine which could do 3,300,000 units of work in one minute, *e.g.* could raise 3,300 lbs. of coal from a depth of 1000 feet in one minute, would be of 100 H.P. It should be pointed out that the stated horse-power of an engine is usually very much below its real horse-power; the nominal horse-power, as it is called, being merely a kind of standard which regulates the price. The amount of work which a man is capable of performing in a given time has also been estimated, but different observers have arrived at very different conclusions, according to the kind of work done. In one case it was ascertained that a man, engaged in raising weights by means of a cord and pulley, did 1560 units of work per minute, or about the $\frac{1}{21}$th part of a horse-power. In the case of men and animals, in addition to the mechanical work (raising weights, &c.) performed, there is a large amount of work done in moving the different parts of the body among themselves, which cannot properly be measured, though it produces fatigue.

EXERCISES.

1. What resistances are overcome in each of the following cases?
   (a) A horse drawing a cart up a hill.
   (b) A horse drawing a boat on a canal.
   (c) A man pumping water out of a well.
   (d) A steam engine drawing a railway train.
2. How many foot-pounds of work are equal to the kilogrammetre, taking the kilogramme as being 2¼ lbs., and the metre as being 39 inches?
3. How much work is done in raising a ton to the height of one yard?
4. A man weighing 130 lbs. walks up a ladder to the height of 30 feet, carrying a hod of mortar weighing 56 lbs., how much work does he perform in the ascent?
5. A horse draws a waggon with a force of half a ton for two miles along a level road; how much work does he do?
6. What power of steam engine would be required to raise 100 tons of coal per hour from a depth of 600 feet, supposing the engine to work continuously?
7. An engine of 100 horse-power is required to pump 3,366,000 gallons of water to a height of 50 feet every day, how long must it work each day? A gallon of water weighs 10 lbs.
8. It is said that a horse walking at the rate of 2½ miles per hour can do 1,650,000 units of work in one hour. What force does he continuously exert?

LESSON 19.

ENERGY.

When a man has done a certain quantity of work he feels weak and tired; in other words, he has lost part of his strength, and has not the same power of doing work as he previously had. If the man is in good health, his capacity for work is restored after he has partaken of food and rest. The term **energy** is employed in Mechanics to denote **the power of doing work**, which is possessed by a man or any other body. A body which does work loses energy in proportion to the amount of work done, and it is this loss of energy which produces the feeling of fatigue in ourselves after having done work.

There are many bodies in the world which have the power of doing work, and by properly making use of these it is possible for man to save himself a vast amount of actual labour. For by means of proper arrangements these bodies may be made to give up the whole, or a part, of the energy which they possess, expending it in doing useful work. Man himself is weak compared with many other animals, but in the end he proves more powerful, since, while they have only their own stores of energy to rely upon, he can make use of the vast stores of energy which other bodies contain. It becomes an important matter, therefore, for man to study carefully the condition of the various bodies around him, and the different forms in which energy occurs, so as to ascertain which of such bodies he can most conveniently employ for the purpose of doing work. The steam in the boiler of a steam engine, a quantity of gun-cotton or gunpowder, a coiled spring, and a stream of water, are all sources of energy, which man can utilise by providing in each case the proper instrument to receive and apply the energy or work-power. These instruments we shall hereafter consider under the name of machines, but for the present we shall confine ourselves to the consideration of the most important forms of energy which various bodies possess.

All bodies which are in motion, and all those which are not in motion but on which some force is acting tending to move them, are capable of doing work. These bodies give out their energy as they move into new positions, if in so moving they have to overcome any resistance, and thus do work. Whenever a body does work it loses energy in proportion to the amount of work done. If it were possible for a body to move from one place to another without encountering any resistance, and therefore without doing any work, it would lose no energy. To determine whether any body possesses energy we must therefore ascertain whether the body is in motion; and, if not in motion, whether it is tending to move under the action of some force, but is prevented by some resistance. In both these cases the body possesses energy; a falling hammer being an example of the one, and a bent bow an example of the other condition. When we speak of the motion of a body, we must be understood to mean either the motion of the body as a whole, or the motion of the molecules of the body among themselves. For, as we shall see, a body may be to all appearance in a state of perfect rest, while its molecules are in reality in rapid motion. It is also important to bear in mind that energy means the *power* of doing work, and not the actual doing of work. A horse in the stable, and a man standing still, may both have the capacity for work in a high degree, and yet at that particular time be in a state of rest.

Force acts in so many different ways that it is usual to distinguish several different forces,—vital force, heat, elasticity, chemical force, &c. In the same way bodies are said to possess different forms or varieties of energy, according to the particular force acting on the body and giving it power to do work. Each of these forms of energy acts in a manner more or less different from the other forms, and requires its own special form of machine or instrument in order to utilise it for the performance of useful work. It will be observed that we do not speak of there being various energies; all energy is alike in being merely the power of doing work. But as we meet with that power in different bodies, we find it manifested in various forms. That we are right in regarding all energy to be one and the same, in spite of the various forms which it takes, is proved by the fact that these various forms of energy can be transformed one into another, as we shall have occasion to point out in the course of our lessons on energy.

Living animals, as the horse, ox, reindeer, &c., are employed for doing work of various kinds,—drawing carts or sledges along the ground, ploughing, raising coal or water, &c. The capacity for work which they possess is plainly due to the possession by them of what we have called "vital force;" and we will therefore speak of that particular form of energy which a living animal possesses, by virtue of its vital force, as **vital energy.** The energy which we ourselves have is vital energy; as is also that of living plants, by means of which they are able to force their roots through the soil, or into crevices of rocks and walls, and by means of which they are able to raise their branches above the ground. There can be no doubt that this form of energy is to a large extent dependent on, and probably the same as, other forms of energy; but for our present purpose it will be best to consider it as a distinct variety of energy, especially as its relations with other forms are not fully and definitely understood.

The energy which drives many clocks is derived from falling weights, which are suspended from cords or chains passing round some of the wheels. These weights are drawn up, that is, raised above the floor of the clock, and then allowed to fall slowly, giving out their energy as they fall. A body of any shape or substance might serve as a clock weight, since all bodies are subject to gravitation, and it is to gravitation that the energy of such raised bodies is due. That form of energy which bodies raised into a position from which they can fall under the action of gravity possess, we shall call the **energy of gravitation.** It must be understood that, whether a raised body be falling or not, it has a capacity for work, so long as it occupies a position from which it is possible for it to fall on account of its gravitation. Water in a cistern on the top of a house, or in a reservoir on the top of a hill; a brick built into the wall of a house; the earth, moon, sun, &c., all of which are urged towards each other by their mutual gravitation, are all examples of bodies having energy of gravitation. Bodies in the lowest position to which they can attain are useless as regards this energy of gravitation. Instead of being able to do work, such bodies, in moving into higher positions, must have work done on them. Water at the bottom of a well, and sand at the bottom of the sea, may be taken as examples of such bodies. In some cases, however, it is worth while expending energy on bodies in low positions, in order to raise them, since they may contain a large quantity of some other form of energy. This is the case with coal. In the succeeding lesson we shall consider other important forms of energy.

EXERCISES.

1. How much work is done in raising an anchor weighing 10 cwts. from a depth of 10 fathoms?
2. A pound of coal when burnt gives out energy sufficient to do about 11,000,000 units of work. Find how much more work would be done (with a perfect engine) by the combustion of one ton of coal, than would be done in raising the coal from a depth of 1,000 feet.

3. Full waggons of coal, running down an incline, are sometimes made to draw the empty waggons up the incline, by means of a rope passing round a pulley. What form of energy is employed in this case? How would you prevent the waggons from running down with too great a speed?
4. Which clock weight will give out the more energy as it falls, one weighing 3 lbs., or one weighing 8 lbs., the distance being the same in each case?
5. Mention several animals employed by man to do work, and describe as far as you can the special circumstances under which each is most valuable.

LESSON 20.

FORMS OF ENERGY.

We have seen in the preceding lesson that energy is met with in various forms; and we there specially considered two of those forms, viz., vital energy, and energy of gravitation. We have now to consider other important forms of energy which are more or less practically useful to mankind.

Fuel is a source of energy of very great practical importance; from it we obtain all the power of our engines. Coal, wood, gas, and the other substances comprised under the term fuel, are all combustible bodies; and they yield up their energy as they burn, just as a raised body gives up its energy as it falls. Chemistry teaches us that burning is really a combination of certain constituents of the fuel with oxygen gas obtained from the air: the substances uniting together in consequence of a certain attraction or affinity between them. This force of attraction we have called chemical force, or chemical attraction; and since it is to this attraction that the bodies owe their energy, we may conveniently term the particular form of energy which they possess **chemical energy**. In the process of burning, this chemical energy is changed into another form of energy, which we call **heat**. After burning, the substances have lost their chemical energy, and are unable to burn again: but from the energy they have lost, a quantity of heat energy has been developed. Gunpowder, dynamite, &c., are bodies possessing this form of energy in a very high degree; a very small quantity of the substance having the power of giving out a large amount of energy.

A stretched spring, or piece of india-rubber, is a body which has the power of doing work, as, for example, closing a door after a careless person. A bent bow has power to propel an arrow when released; and a coiled watch-spring has power to set in motion, and keep in motion, the works of the watch. It is obvious that the power in such cases is due to the force of elasticity, acting to bring these bodies back to the condition most natural to them, after they have been distorted. And the form of energy which they possess we may therefore speak of as the **energy of elasticity**. Compressed air, which is now frequently employed to work engines in positions where steam engines would be unsuitable, is a body which owes its energy to elasticity. Such engines are employed in coal mines for cutting coal, and in tunnels for boring: places where the escaping steam and smoke would be inconvenient, but where the escaping air is of great service in aiding ventilation.

It is very evident that there is a considerable difference between a cannon-ball at rest, and the same ball moving at the rate of several hundred feet per second. The ball might have practically no energy when at rest, but everyone will allow that it possesses a considerable amount of energy when in motion, and that its energy depends entirely upon its motion. A hammer resting upon a nail has little power to force the nail into the wood, but the hammer in motion has its store of energy largely increased, the extra energy being entirely due to the motion. Moving bodies, therefore, possess energy on account of their motion, and such energy has been called **mechanical energy**. We have, in the case of wind, and streams and currents of water, familiar natural examples of bodies having mechanical energy. The wind is nothing more than a body of air in motion; and its energy has long been utilised, by means of wind-mills, for grinding corn, pumping water, &c. In the case of sailing ships, the mechanical energy of wind has also been employed to impart motion to the vessel, in spite of the resistance of the water. The energy of moving water has been similarly utilised by making it turn round a properly constructed wheel, with which is connected apparatus fitted for grinding corn, or for some other useful purpose. These two natural sources of energy to which we have just referred have been somewhat neglected in recent years in favour of coal. But when our supplies of coal begin to run short, as they must do at no very distant period if we go on at the present rate of consumption, the energy of wind and water will again become most important to us.

Heat is a form of energy of extreme importance to us. We have already seen that much of this energy which we make use of is obtained by the combustion of fuel. But in the sun we have a vast store of heat, of which, however, only an exceedingly small proportion of the total quantity radiated from the sun reaches our earth. The sun's heat evaporates the water of the ocean, and the vapour so formed, after passing into the air, is sooner or later condensed, and falls as rain, part of which goes to form those streams whose mechanical energy we have just considered. The expansion of the air by heat in certain parts of the earth starts those currents of air, or winds, which, as we have seen, are sources of energy.

Light is also a form of energy which is in many ways closely related to heat. This energy, though not perhaps of such direct importance to man as heat, performs a great work in the world; for it is the energy which enables plants to grow and build up their substance, thus forming for man both his fuel and his food.

And, lastly, both magnets and electrified bodies are sources of energy. A stick of sealing wax, which has been rubbed with a piece of flannel and thus electrified, will produce motion in light bodies, such as pieces of paper or straw. Electricity, obtained in a somewhat different manner, is employed to move the needle of a telegraph instrument, or to produce the intense heat and light of the electric lamp. We may conveniently refer to this form of energy as the **energy of electricity**. The **energy of magnetism** is very closely related to that of electricity, so much so that it is very probable that they are one and the same; but as a matter of convenience we will speak of them by different names. The greater part of the electricity now employed for lighting and other purposes is produced by means of magnets. Electricity is a force which is becoming of more importance year by year, and there can be no doubt that in the future many operations which are now performed by means of some of the other forms of energy, will be more conveniently performed by means of electricity.

We may now sum up the various forms of energy to which we have very briefly referred. To have described these various forms, even briefly, would have carried us very far beyond our limits. A fuller knowledge of their characters would be highly interesting, as well as very important, but it must be obtained from special works on the various subjects. The forms of energy referred to in this and the preceding lesson may be enumerated as follows:—(1) Vital energy, or the energy of living things; (2) chemical energy, or the energy of chemical affinity; (3) the energy of gravitation; (4) the energy of elasticity; (5) mechanical energy, or the energy of moving bodies; (6) the energy of heat, and of light; and (7) the energy of electricity, and of magnetism.

EXERCISES.

1. What form of energy is possessed by each of the following bodies:—steam; an avalanche; coal gas; a water fall; a "head of water;" and a fire?
2. What reason is there for believing that the heat which falls on the earth is only a very small part of that which is given out by the sun?
3. When an arrow is shot up in the air, what form of energy has it during its upward flight, and at its highest point respectively?
4. Mention some practical applications of the energy of elasticity, and of the energy of heat.
5. Has a rifle bullet as much energy when it strikes the target as when it left the gun? What work does it perform in its flight?

LESSON 21.

KINETIC ENERGY.

When we consider the various forms of energy enumerated in the last lesson in connection with each other, we perceive that it is possible to arrange them into two distinct classes. Some of the forms of energy—mechanical energy, for example—are evidently due to the motion of the bodies; and are lost to the bodies, as such, when their motion is destroyed. Other forms of energy are just as plainly due to the position in which the bodies are placed, and to the position only; that is to say, the energy is independent of the state of the body, whether of rest or motion. The energy of gravitation possessed by an elevated body is of this latter kind, as is also that possessed by a piece of iron in the neighbourhood of a magnet; the energy in both cases being due to the relative positions of the bodies. We therefore regard energy as being of two kinds, viz., (1) energy due to motion, or, as it is usually called, **kinetic energy**, and (2) energy due to position, usually called **potential energy**. We must endeavour to understand clearly the essential difference between these two kinds of energy; and, in order to do that, it will be well for us to consider a few examples of bodies possessing each variety. In the present lesson we will confine ourselves to kinetic energy.

The energy which gives a cannon ball the power to pierce an iron plate, or a great thickness of wooden planks, is obviously due to its motion. The same ball might rest for an indefinite time against the bodies without penetrating them in the least. It is only a rapidly moving ball which can overcome the résistance opposed by the cohesion of the bodies. As the ball penetrates the plates or planks farther and farther, its velocity is rapidly diminished, and its energy (and therefore its penetrative power) becomes less and less, until at last it is brought to rest, with all its mechanical energy gone. Since the ball has mechanical energy only when in motion, and the amount of energy depends upon the quantity of motion, we are justified in assuming that the energy is due to the motion. In the same way the energy by means of which a hammer can drive a nail, or flatten a piece of lead, is due to the motion of the hammer; the mere weight of the hammer at rest would be insufficient to move the nail or the molecules of the lead. But, partly by the action of gravity, and partly by the muscular force of the person using the hammer, it is set in more or less rapid motion; and in this condition it has a considerable amount of energy. The energy by means of which a train, approaching a station with the steam shut off, is able to overcome the resistance of the air, and of friction against the rails, is entirely due to the velocity produced before the steam was shut off. The quantity of energy possessed by the train diminishes as the resistances lessen its velocity; until, when it is brought to rest, it has no mechanical energy at all. Mechanical energy is therefore one form of kinetic energy.

A body may not be in motion as a whole, and yet the various parts of the body may be moving amongst themselves. There are good reasons for believing that heat is nothing more than a state of motion of the molecules of a body; and that the difference between a hotter and a colder body is due to the difference in the velocity with which the molecules are moving. The rapidity of the motion of the molecules in the hotter body is greater than in the colder body; and, therefore, the energy of the hotter body will be greater than that of the colder body. If we could deprive the molecules of a body of all motion, we should also deprive the body of all heat energy. If heat be really of this nature, we must regard the energy of heat as another form of kinetic energy.

Mechanical energy and heat are the most familiar forms of kinetic energy; but there are doubtless other cases of energy due to motion, and therefore to be classed as kinetic. The energy of a compressed gas, which is believed to be due to a motion of its particles, and the energy of electric currents—such as those employed in telegraphy—are probably kinetic. The motion in these cases, as in that of heat, is molecular, and very little is definitely known about it.

We must now consider the conditions on which the amount of kinetic energy possessed by a body depends. In the first place, it depends upon the mass of the moving body—the greater the mass, the greater the amount of kinetic energy, other things being equal. A heavy hammer will strike a harder blow, and produce a greater effect, than a lighter one, though both may be moving with exactly the same velocity. Of two railway trains moving with the same velocity, the more heavily loaded train—that is, the one having the greater mass—will do the more damage in case of a collision. A cannon ball weighing 1,000 lbs. will do as much work before coming to rest as two balls of 500 lbs. each, moving with the same velocity, and, therefore, will do twice as much work as one ball of 500 lbs. The kinetic energy of a body, therefore, varies in the same proportion as its mass varies.

But the kinetic energy depends also upon the velocity with which the body moves. Everyone knows that the same hammer can be made to strike blows having very different degrees of intensity. To strike a powerful blow the hammer is made to move with a greater velocity than when it is desired to strike a less powerful blow. The greater the velocity with which a body is thrown upwards, the greater the height to which it will rise, and therefore the greater the amount of work done in raising it. It is found by experiment that a body thrown upwards with a velocity of 32 feet per second rises to a height of 16 feet; while a body started with a velocity of 64 feet (or twice as great as before), rises to a height of 64 feet, that is, to a height four times as great as that in the former case. Doubling the velocity, then, increases the amount of energy fourfold; making the velocity three times as great as at first would increase the energy nine times; and, as a general rule, the energy increases as the square of the velocity.

The kinetic energy of a body, therefore, varies in the same proportion as its mass and as the square of its velocity. Thus, suppose we have a body with a mass of 100, and moving with a velocity of 10 ft. per second. If the mass is increased to 200, and the velocity remains the same, the kinetic energy will be twice as great as before. But if the mass remain 100, and the velocity be doubled, the energy will be four times as great as at first. While if both mass and velocity be doubled the energy will be 8 times as great as it was originally. To increase the quantity of kinetic energy, therefore, a greater advantage is gained by increasing the velocity, than by increasing the mass in the same proportion.

According to the first law of motion, a moving body which encounters no resistance continues to move uniformly with the same velocity, and therefore its store of kinetic energy remains the same. But if the body meets with resistance, it expends a portion of its energy in overcoming this resistance, and its store is diminished. Every body in motion has a certain store of kinetic energy, which it will spend in doing work before the body can be brought to rest. It is important to be able to calculate the amount of work which any body in motion will do before it comes to rest, and this may be done by means of the following rule:—Multiply the mass of the body (in pounds) by the square of the velocity (in feet per second), and divide the product by 64; the answer will give the number of units of work which the kinetic energy of the body can accomplish. As an example, take the case of a ball weighing 160 lbs., and moving with a velocity of 200 feet per second, then

$$\frac{160 \times 200 \times 200}{64} = 100{,}000$$

or the ball will do 100,000 units of work before coming to rest.

EXERCISES.

1. One body has a mass of 10, and a velocity of 15 units; another body has a mass of 15, and a velocity of 10 units. Which body has the more kinetic energy?
2. A body weighing 4 lbs. is projected upwards with a velocity of 32 feet per second; to what height will it rise, and how many units of work will it do in raising itself?
3. A cannon when fired moves backwards with the same momentum as the ball moves forwards; is the amount of energy the same in the two cases? If not, which body has the more energy?
4. A mass of iron weighs 50 lbs., and moves with a velocity of 8 feet per second; how much work will be done in stopping it? If the velocity were increased to 12 feet per second, how much more work would be done than in the former case?
5. If a large wooden ball and a small wooden ball were set in motion with the same amount of force, how is it that the large ball would move more slowly and stop sooner than the small one?

LESSON 22.

POTENTIAL ENERGY.

When a piece of iron is in contact with a magnet, there is a certain amount of attraction between them, by reason of which a resistance is opposed to any force tending to separate them from each other. To remove the iron from the magnet work must be done, the amount of work depending upon two things, viz., the intensity of the force of attraction between the bodies, and the distance from the magnet to which the iron is removed. The attraction between the magnet and iron exists when they are separated, and were all resistances, such as friction, removed, the two would move towards each other. In the course of their motion towards each other, either, or both, of them might be made to do work,—to raise a small weight, for example. Thus a piece of iron in the neighbourhood of a magnet is a body which possesses energy, and which, under certain circumstances, may be made to do work. Its energy is evidently due to the action of the force which we have called magnetism. But this force still acts when the iron is in contact with the magnet; yet in that position neither the iron nor the magnet is capable of doing work. In order to have the power of doing work it is necessary that a body should be in a position from which it can move. The difference between the iron in contact with the magnet, and the iron separated from the magnet, is a difference of **position**; and the energy which it has in the latter case, and has not in the former, is due to the position of the iron with reference to the magnet. In other words, energy of magnetism is a form of potential energy, or energy which bodies have on account of the position in which they are placed with respect to certain other bodies.

The same is true of the energy of gravitation. A body may be able to do work owing to its gravitation, or the attraction existing between it and the earth. But, that the body may have the power to do work on this account, it is an essential condition that it must be elevated, or in a position from which it can move towards the earth. A clock weight is powerless to move the wheels so long as it rests on the floor of the clock case, or hangs at the end of the cord which is unwound to its full extent. Under these circumstances, although gravitation acts as much as ever, there is no possibility of movement, and therefore no available energy. But when the string is wound up, and the weight raised to a position from which it can move downwards when the pendulum is started, it possesses a store of potential energy. A tank of water on the top of a house is a store of potential energy, which can be utilised by suitable arrangements for the performance of various useful operations. But if the water is stored in a tank at the bottom of the house, it will be no longer available as a source of energy, although precisely the same as in the first case in every respect except that of position. Bodies on the surface of the earth, or, rather, in the lowest positions to which they can attain, must have energy expended on them to raise them, before they can themselves be in a position to do work by virtue of their gravitation.

It will be observed that in the case of the potential energy of gravitation, as well as in that of magnetism, a body possessing the energy actually does work only when it moves. That is to say, a body may possess the power or capacity for work while at rest, but in order that it may do work it must actually move. **Potential energy must be converted into kinetic energy in order to do work.** The fact that a body may retain its potential energy unchanged for any length of time, constitutes one of the most important practical differences between kinetic and potential energy. Bodies in motion soon come to rest, as a rule, in consequence of the various resistances which they encounter ; and therefore, if we would utilise the kinetic energy of a body in motion, it is necessary to do so at once. But we may store up, as it were, potential energy for any length of time, and use it when most convenient to our purpose. A wind-mill, or a sailing vessel, moved by the kinetic energy of the wind, must be ready to make use of the wind when it may happen to blow. But a steam ship, or a mill worked by means of a steam engine, which derives its power from the potential energy of coal, may work or wait to suit the convenience of the master, without fear of the energy being wasted.

The potential energy of coal, and fuel generally, appears to depend upon the position of various substances with regard to each other. It would require a considerable knowledge of chemistry to understand this matter fully ; but we may say generally that the energy of combustion is due to the uniting together of certain substances, which were previously separated from each other. Carbon, and other substances in the fuel, have an attraction or affinity for the oxygen in the air, just as a magnet and a piece of iron attract each other. On account of this chemical attraction, the carbon and oxygen tend to move towards, and unite with, each other, producing by their combination the kinetic energy of heat. While still separated from each other these substances possessed energy of the potential variety ; but when combined together to form carbonic acid gas, they have no available energy, since they have no further possibility of movement. We must therefore regard the energy of chemical affinity or chemical attraction as potential energy.

The energy of a wound spring, and of distorted elastic bodies in general, is potential energy, due to the relative positions of the molecules of the bodies. In altering the form or size of the body we disturb the equilibrium of the molecules, moving some of them nearer together, and perhaps also, at the same time, separating some farther from each other. These new positions are not positions of equilibrium, so that as soon as the distorting force is removed the body recovers its original size or shape more or less perfectly, and during the recovery may be made to do work. Here, again, it will be observed that the potential energy (of elasticity) must be converted into kinetic energy in order to do work.

When two bodies have been rubbed together, so as to become electrified, there is an attraction between them. In separating two such bodies from each other work is done, and when they are separated they both possess potential energy, on account of their tendency to move together under the influence of this electrical attraction. We may therefore consider this as the potential energy of separated electrified bodies.

We have now enumerated the chief forms of potential energy, and pointed out that all agree in the fact that the energy is due to the position of certain bodies with regard to each other, or to the relative positions of the molecules of a body. In every case the bodies which possess the energy are tending to move because of some force acting on them: and they can only do work as they move. The amount of potential energy which a body possesses depends partly upon the magnitude of the force acting on the body, and partly upon the distance through which the body would be able to move. The potential energy of a raised body, for example, depends upon its mass (the amount of gravitation being in proportion to the mass), and upon the height to which it is raised.

EXERCISES.

1. Give an example of a body which possesses both kinetic and potential energy at the same time.
2. In some operations a very powerful force is required at intervals, and it is obtained from the descent of a heavily-loaded cylinder, which, during the intervals, is raised by a small hydrostatic press kept continually at work. What advantages are there in such an arrangement?
3. What advantages are there in placing reservoirs of water for the supply of towns in elevated positions? How is the water raised into these reservoirs?

LESSON 23.

CONSERVATION OF ENERGY.

We have had occasion, in previous lessons, to speak of the various forms of energy as being convertible one into another, and of potential energy being always transformed into kinetic energy in order to do work. In connection with these changes there are some facts and principles of the highest importance, which we will now proceed to consider.

The bob of a pendulum at rest has no kinetic energy; and so long as the string to which it is suspended remains unbroken, we may consider it as having no potential energy. In order to set the pendulum in motion, a certain amount of work has to be done, and energy expended, on it. In consequence of the energy transferred to the pendulum, it commences to rise, having at the beginning of its motion a certain amount of kinetic energy. As it rises its velocity diminishes, and therefore its store of kinetic energy becomes less, until, at the highest point of its path, it has no kinetic energy at all. But while it has been losing kinetic energy it has been gaining potential energy; and the amount of potential energy which it possesses is greatest at the time when its kinetic energy is least—viz., at its highest point. So that during its rise the pendulum has exchanged its kinetic for potential energy: the former variety having, so to speak, been transformed into the latter. In the course of the descent, the opposite change will take place—potential energy being gradually converted into kinetic energy; and as the pendulum passes through its lowest point, its energy will be entirely kinetic. In the absence of the resistance of the air, and of friction against the supports, the pendulum would commence its second ascent with exactly the same velocity (and, therefore, the same amount of kinetic energy) as that with which it started in the first instance, and it would rise to the same height as before. And this would go on for any length of time in the absence of resistance; so that, after a thousand or a million swings, the velocity with which the pendulum passed through its lowest point would be precisely the same as the original velocity imparted to it, and the height to which it would rise would be the same as that to which it rose in its first ascent. In other words, the amount of energy imparted to the pendulum in the first instance would remain unaltered in amount, provided the pendulum had no work to perform in overcoming external resistance. This energy would not, however, be of precisely the same kind for any two consecutive instants. At one instant it would be all kinetic, at another all potential, while during a great part of the time it would be partly kinetic and partly potential. The form of the energy, therefore, would change over and over again, but throughout all the changes there would be neither destruction nor creation of energy, the total quantity remaining constantly the same.

In the case of an ordinary pendulum, which has to perform work in overcoming the resistances opposed to its motion, the height to which it rises, and the velocity with which it moves, diminish with each swing. The energy of the pendulum therefore gradually grows less; but a little consideration will show us that this energy is not destroyed, but is transferred from the pendulum to the bodies on which it does work. The air, for example, which impedes the motion of the pendulum, is somewhat heated by the friction, and is also set in motion by the pendulum, thus gaining energy in two ways. The energy gained by the air is derived directly from the pendulum, and thus the amount possessed by the latter is diminished. The friction of the pendulum against its supports also involves a loss of energy to the pendulum. But this friction is the means of producing heat, and therefore the energy is only lost in one form to be gained in another. In this case, as in the case of a perfect pendulum, there is neither creation nor destruction of energy; but simply a redistribution of the original energy, together with more or less change in its form.

This principle, which is known as the **conservation of energy**, is found to be true in every case. Energy can neither be created nor destroyed, though it may be transferred from one body to another, and may take any of the various forms in which energy is found to exist. When one body gains energy, it is at the expense of some other body, which has lost an equal amount; when a quantity of energy disappears in one place, an equal amount of the same or some other form of energy appears in another place; so that the total quantity of energy in the world remains constantly the same. This principle of the conservation of energy, which is one of the most important principles in natural science, may be stated in the following form :—**The various forms of energy may change one into another, but the total amount of energy in the universe is neither increased nor diminished by any number of such changes.** The truth of this principle, like that of the laws of motion, is not capable of direct proof; but from the vast number of cases in which it has been found to accord with the results of observation and experiment, and in the absence of all evidence to the contrary, there can be no reasonable doubt that the principle is true.

We have, in a previous lesson, described a piece of iron in contact with a magnet as having no potential energy of magnetism, and have stated that the same piece of iron when removed from the magnet does possess such energy. Whence is this energy derived? To separate the iron from the magnet work has to be done; and the body which does the work expends a certain amount of energy in doing it. The energy so lost by the body doing the work is gained by the magnet and iron, with the exception of any that may have to be spent in overcoming resistances other than that due to the attraction between the magnet and the iron, such, for example, as friction or the resistance of the air.

Energy must similarly be expended in doing the work of compressing or coiling an elastic spring, in stretching a piece of india-rubber, or in bending a bow. Such energy is not destroyed, but is stored up as potential energy in the distorted body. The very force which causes the resistance to the distortion, and which therefore necessitates the expenditure of energy, is the force by virtue of which the bodies tend to recover their original condition, and thus derive their capacity for doing work. The greater the amount of energy expended on the elastic bodies, the greater will be the amount given out by them afterwards. The energy spent in driving the works of a clock, or watch, is the energy which was expended in winding up the spring.

If we make allowance for the resistance of the air, we find that the velocity acquired by a body falling from any height, is exactly the same as the velocity with which it must be projected to enable it to rise to that height. If a stone is projected upwards with a given quantity of kinetic energy, it does work in raising itself to a certain height, and gradually exchanges its kinetic energy for an equivalent quantity of potential energy. On falling to the level from which it started, it will acquire the same velocity, and therefore possess the same amount of kinetic energy, as that originally imparted to it. A body, in falling from a certain height and encountering resistance, is able to do as much work as was done on the body to raise it to that height. Thus, a weight of 12 lbs. raised to the height of five feet would involve the expenditure of 60 units of work, and would have the power of doing 60 units of work in falling to the ground. The kinetic energy which a body falling freely has on striking the ground is apparently lost by the body being brought to rest. But it appears that a quantity of heat is developed by the blow, and that the quantity of heat-energy produced is precisely equivalent to the mechanical energy lost. Thus there is a change in the form, but not in the amount, of energy.

In the case of a fire, the substances formed by the burning of the fuel have no longer affinity for oxygen; that is to say, they have lost their potential energy of chemical affinity. To obtain a quantity of the energy of heat, it is absolutely necessary to sacrifice an equivalent quantity of some other form of energy; and in the case of a fire the form of the energy which disappears is chemical energy. The enormous gain of mechanical energy gained by the explosion of gunpowder is accompanied by an equally large loss of chemical energy. The heat and vital energy of our own bodies is obtained at the expense of a certain amount of chemical energy contained in our food. Fat and sugar, two of the substances from which we obtain a great part of our energy, are bodies which will readily burn; and in our bodies they are converted into the same substances as they would be in an ordinary fire.

We might go on to a very great length pointing out instances of the transformation of energy from one form into another, and illustrating the general principle of the conservation of energy. Every phenomenon or occurrence which takes place is the result of the transference of energy from one body to another, or of a transformation of energy from one form to another. And in every case without exception it is found, when the circumstances are fully known, that the principle of the conservation of energy holds good.

EXERCISES.

1. Springs are frequently attached to doors to close them after careless people. Where does the energy which closes the door really come from?
2. When a man walks up a ladder, has he as much energy at the top as he had at the bottom? Will he have as much energy on reaching the ground again as when he started? What changes of energy take place during the operation?
3. Explain as fully as you can the changes of energy involved in the striking of a match.
4. A ball of clay weighing 1 lb., and suspended as a pendulum, strikes a similar and similarly suspended ball with a velocity of 10 ft. per second. The mass of clay weighing 2 lbs. moves, after the collision, with a velocity of 5 ft. per second. Is the amount of kinetic energy the same after as before the collision? If not, can you explain what has become of that which has disappeared?

LESSON 24.

NATURAL SOURCES OF ENERGY.

The principal natural sources of energy available to man for the purpose of doing work, have been shortly summed up as follows:—Fuel, Food, Wind, Rain, and Tides.

Under the head of **fuel** we include all substances practically used, or capable of being used, for the production of heat by their combustion, such as coal, wood, peat, petroleum, &c. Of these we must regard coal as by far the most important. Peat and wood are both being formed in considerable quantities at the present day, in certain countries of the world; but if we were obliged to rely on these two sources for our fuel, they would soon be exhausted. In our coal-fields, however, we are fortunate in possessing vast stores of energy, in a most convenient form. Coal, we are told, has been formed from the vegetation which existed on the earth countless ages ago; and has been preserved and brought into its present form by being buried under immense beds of rock. By means of energy derived from the sun, plants are enabled to take from the air certain substances, which they store up in their structure, chiefly in the form of wood. Some of the energy radiated from the sun and falling on the earth is thus appropriated by plants; and the energy we obtain by the combustion of coal, or wood, or other vegetable matter, is in reality energy derived in this manner from the sun. The energy is stored up in the plant as potential chemical energy; in order to utilise it, we convert it into the kinetic energy of heat. In the light of what we have just stated, we may understand the remark, said to have been made by George Stephenson, that it was the sun which drove his engines. And we are also in a position to understand the statement frequently made, that the light and heat of our fires and gas flames, is really the light and heat emitted by the sun in those ages long ago, when the plants grew which formed our coal. One of the chief sources to which we owe our commercial and manufacturing prosperity, is the abundance and convenience of our coal supplies. Yet when we consider that from our English coal-fields, which are by no means unlimited in extent, no less than 150,000,000 tons are now extracted every year, we shall see that, slowly perhaps, but surely, the time will come when our coal-fields will be practically exhausted. And as the difficulty of obtaining coal, and therefore the expense of it, increases, we shall be driven to make more extensive application of the other natural sources of energy than we do at present.

Food, like fuel, possesses potential energy derived from the sun. Our vegetable food (wheat, potatoes, &c.,) is formed by the plant by virtue of the sun's energy, just as we have explained the woody part of the plant to be so formed. And the energy comprised in our animal food (meat, milk, &c.), is merely energy which the animal, in its turn, obtained from the vegetables on which it fed. So that ultimately we may trace back the energy of both food and fuel to the sun, and may look upon plants as so many workshops in which this energy is being collected, prepared, and stored up, ready for the use of ourselves and other animals.

The energy of the **wind** is kinetic, being due to the motion of the atmosphere in relation to the surface of the earth. It has long been utilised for propelling ships, and was formerly much more generally employed by means of windmills for pumping water, grinding corn,

&c., than it is at the present time. It is a cheap, and sometimes very convenient, source of energy; its great drawback being that it is so irregular in its action. This objection may, however, be overcome to a great extent, by employing wind-power, not to do the required work directly, but to store up potential energy in some form or other, which store may be drawn upon as found most convenient. Under these conditions it is not unlikely that the wind may at some future time again become one of our chief sources of energy. Like the energy of food and fuel, the energy of the wind is derived from the sun; the currents of air being set in motion by the unequal heating of the atmosphere in different parts of the world by the sun's rays.

The **rain** is given as a natural source of energy, since by its means are formed rivers, lakes, and other bodies of water in more or less elevated positions, and, therefore, possessing more or less of the potential energy of gravitation. A portion of the waters on the surface of the earth is converted into vapour, which rises into the atmosphere, in the higher regions of which it becomes condensed again into the form of water. This water is in a position of advantage as regards the earth, and in its descent may be made to yield the energy it possesses for the purpose of doing work. The rain which falls on the hill-sides and forms streams may be, and frequently is, made to turn a water-wheel, and thus expend a portion of its energy in the useful work of grinding corn, instead of wasting such energy in friction against the river-banks and the stones in its course. The enormous amount of energy present in the case of waterfalls will, doubtless, some day be made in part available for the use of man, being converted into electricity, or into some other form capable of transmission to a distance. In the case of the Falls of Niagara, where it has been estimated that, on the average, no less than 700,000 tons of water fall every minute from a height of 160 feet, what an enormous amount of mechanical work the energy expended in the fall would do, if some means could be devised for taking advantage of it! It is obvious that in the case of rain, as in each of the preceding sources of energy, the energy is primarily derived from the sun.

The energy of the **tides** is the only practically important source of energy available on the earth which is not derived directly or indirectly from the sun. It is in fact derived from the rotation of the earth on its axis, the velocity of rotation being slowly diminished by the friction of the tidal wave. The subject, however, is a very difficult one to understand fully, and it must be sufficient in these lessons to merely state the fact. The energy of the tides is but little utilised at present; and it is doubtful whether the tides will ever become a profitable source of energy to any very considerable extent. The manner in which it might be possible to utilise the tides as a source of energy is not difficult to understand. A number of large reservoirs or basins might be so constructed as to become filled at high water, and the water retained when the tide ebbed. Then at low water the contents of these reservoirs might be allowed to escape, and the water made to do work in falling to the lower level.

There are several other sources of energy which might be mentioned, but they are altogether insignificant in their practical bearings in comparison with those mentioned above. It will be seen, therefore, that almost all the energy, or power of doing work, which we possess on the earth, is energy which comes to us from the sun. We have seen that the sun is the great central mass round which the earth and the other planets revolve, and by whose attraction they are retained in their orbits. We regard the sun as our great source of light and heat, and now we must also regard him as our great source of power. The energy which the sun now yields in such enormous quantities, it must itself have derived from some source or other, and what this source may be is a most interesting subject of enquiry, but one into which we cannot attempt to enter in these lessons.

EXERCISES.

1. Taking the figures given in this lesson, calculate what horse-power is equivalent to the energy of the Falls of Niagara?
2. Can you suggest any arrangement by means of which a wind-mill could be made to store up potential energy?
3. Mention as many substances as you can which are practically used for fuel. What do you consider are the characters of a good fuel?
4. What are the tides? How are they produced? What interval of time elapses between two successive times of high water at a given place?

LESSON 25.

HEAT AND MECHANICAL ENERGY.

When our hands are cold we very frequently rub them together in order to warm them, the friction between them being in some manner or other capable of producing heat. Boys sometimes rub a metal button on a piece of wood or on their clothes, and by that means make the button so hot that they can scarcely hold it. When a heavily laden cart is taken along a rough road, having one wheel fastened so that it cannot turn, sparks are struck from the stones by the friction of the wheels. These sparks consist of small fragments of stone, which are rendered red hot or white hot as they are struck off. And, in the same way, when a train approaches a station with the break applied, sparks may be seen to fly off as the wheels slide over the rails. The production of fire by savages, by rubbing together rapidly two pieces of dry wood, is another fact of the same kind.

There appears to be, therefore, a close connection between motion and heat, or between the expenditure of mechanical energy and the development of that form of energy which we call heat. And a little observation will show that the amount of heat produced is proportional to the amount of mechanical energy expended; the production of heat being most striking when there is most friction, and, therefore, when most energy is spent in producing motion. The connection between these two forms of energy has been very closely studied in recent years; and it has been conclusively proved that, in cases such as those referred to above, there is a direct conversion of motion, or mechanical energy, into heat. Heat, as we have stated in a previous lesson, is believed to be due to the motion of the molecules of bodies, the intensity of the heat being in proportion to the velocity with which the molecules are moving. When two bodies are rubbed together, and there is friction between them tending to prevent their movement, the motion produced will not be smooth and uniform, but jerky and irregular. And we can roughly picture to ourselves how this irregular motion will agitate the molecules of both bodies; just as when a box full of stones is dragged over a rough surface, the stones are rattled and shaken one against another.

This subject of the conversion of mechanical energy into heat is so important, that it will be well for us to consider a few more instances in which the change occurs. Sir Humphry Davy found that he was able to melt two pieces of ice by simply rubbing them together; taking care, of course, to prevent heat passing from his hands or other source to the ice. The "shooting stars," which we may sometimes see at night, are not stars at all, but comparatively small bodies drawn to the earth by gravitation. It is believed that the heat which makes them hot and luminous is produced by friction against the air, moving as they do with an almost inconceivable velocity. And it is further believed that, in most cases, the heat produced by the friction of these bodies against the air is so intense as to convert them into vapour; and thus a vast number of them never reach the earth at all. By hammering a small piece of lead it can easily be made too hot to be held in the hand, and a blacksmith will keep a piece of iron red hot for a long time by the heat derived from the skilful blows of his hammer. A saw which has been used to cut through a piece of wood, or a gimblet or augur used to bore through the wood, is found to be considerably heated when withdrawn. And this is especially the case when the wood is very hard; the great amount of energy required to move the tools resulting in the production of a correspondingly large amount of heat. For the same reason more heat is developed in the working of metal than in the case of wood; and on this account it is usual to let water continually drop on the hard steel tools employed to plane or bore the metal, in order to prevent them from being heated so much as to destroy their hardness.

Nearly 100 years ago Count Rumford carried out a number of experiments, in connection with the production of heat in the working of metals; and by these experiments he satisfied himself that heat is not a substance, as was generally believed at that time. In the account which he has himself written of his experiments, he describes how, while he was engaged in superintending the boring of brass cannon at Munich in Bavaria, he "was struck with the very considerable degree of heat which a brass gun acquires, in a short time, in being bored, and with the still more intense heat (much greater than that of boiling water) of the metallic chips separated from it by the borer." He arranged a special experiment, in which the heat produced by the friction of a hard steel borer against a brass cylinder was given up to two and a half gallons of water, with which the cylinder was surrounded. And he goes on to express the pleasure it afforded him, and the astonishment it caused in the bystanders, to see at the end of two hours and a half this large quantity of water commence to boil, the heat having been entirely produced by the conversion of the mechanical energy expended by the two horses which turned the boring tool. The heat produced by the friction of the axles of railway carriages against the axle boxes, has more than once proved sufficient to set the carriage on fire. Hence it is necessary that the axles and bearings should be very carefully made, and that there should be a liberal supply of grease or other lubricating substance, so as to reduce the friction to a minimum.

As examples of the conversion of heat into motion we may refer to the following facts. The general effect of heat on bodies is to alter their size; an increase of temperature producing in almost every case an increase of size. This alteration of size implies motion on the part of the molecules of the body. Again, in the conversion of water into steam, a large quantity of heat passes into the water which does not raise its temperature; being altogether employed in overcoming the cohesion of the liquid, and moving the molecules so far from each other as to be out of the range of each other's attraction. It has been proved by direct experiment that a quantity of heat is lost as such on the working of an engine. That is to say, the amount of heat which escapes from the engine, along with the escaping steam, hot gases from the fire, &c., is less than the quantity produced by the combustion of the fuel. And we have every reason to believe that the heat so lost has been converted into the mechanical energy which the engine has expended on the work it has done. When air is compressed heat is produced, the energy spent in moving the molecules nearer to each other being converted into heat. On the other hand, when compressed air is allowed to expand it becomes cooler, a portion of its heat being converted into motion. This cooling, produced when air is released from a state of compression, forms a serious difficulty in the employment of engines worked by means of compressed air; the cylinders and tubes in which the air expands becoming so cold, as not only to condense on themselves moisture from the air, but even to freeze it.

Very careful experiments have been made with a view to determine the "mechanical equivalent of heat," or, in other words, what amount of mechanical energy is equivalent to a given quantity of heat. From a large number of experiments performed by Dr. Joule, of Manchester, it is concluded that if a quantity of water were to fall from a height of 772 feet, and the heat produced by its striking against the ground were all taken up by the water, the temperature of the water would be raised one degree. On the other hand, if the heat required to raise the temperature of any quantity, say a gallon or 10 lbs., of water, were spent instead in lifting it, it would be sufficient to raise the water to a height of 772 feet. If we call the quantity of heat required to raise the temperature of a pound of water one degree (according to the ordinary Fahrenheit scale), one unit of heat; then we may say that one unit of heat is equivalent to 772 units of work. For that quantity of heat would raise the pound of water (or a pound of any other substance), to a height of 772 feet, or would raise 772 lbs. to a height of 1 foot, if it were all converted into mechanical energy. From this it may be calculated that the quantity of heat spent in melting 1 lb. of ice, and converting it wholly into steam, is equivalent to nearly one million foot-pounds of work.

EXERCISES.

1. Explain the origin of the sparks when a flint is struck against a piece of steel.
2. Mention some special examples of the expansion of bodies by heat.
3. Show how the heat of a fire is the cause of the motion of the air in the chimney.
4. When a hammer strikes a nail, what becomes of the energy of the hammer?
5. When a cannon ball moving with great velocity strikes an iron target, a flash of light is seen. Explain, as far as you can, the cause of this.
6. A sounding bell or tuning fork is in a state of rapid vibration, and therefore possesses mechanical energy. What has become of such energy when the sound has died away?

LESSON 26.

THE NATURE OF HEAT.

For a very long time it was believed that heat was a material substance, or form of matter, associated in variable quantities with the ordinary matter of which bodies are composed. A body when hot was believed to contain a larger quantity of **caloric** (as this heat-substance was called) than when cold. From hot bodies, such as the sun, or a fire, streams of particles of caloric were assumed to be continually passing off, and travelling through space with a very great velocity. But within the last hundred years this material theory of heat has gradually been altogether abandoned, in favour of what is called the "mechanical theory of heat," which explains heat as being due to molecular motion. The most careful experiments have failed to show that there is any difference between the weight of a body when hot and when cold. But it was chiefly by the consideration of the relations between heat and motion, such as those referred to in the last lesson, that scientific men were led to regard heat as a mode of motion. The experiments of Davy and Rumford are usually regarded as conclusive in proving that heat is not a material substance, whatever else it may be. As was pointed out by Rumford, an unlimited quantity of heat might be obtained by the friction of two bodies, provided sufficient time were given. But, if heat is a substance, there can only be a limited quantity of it present in the two bodies, and we cannot conceive of any process by which it would be possible to convert a limited quantity of matter into an altogether unlimited quantity. We are therefore driven to adopt some other theory of the nature of heat. And since the theory that heat is a kind of molecular motion explains all the phenomena connected with, and produced by, heat, we may accept it as true.

That heat is closely connected with motion we have shown in the last lesson, by describing numerous instances where heat is converted into motion, or motion into heat. The radiation of heat by hot bodies, such as the sun, is evidently the result of motion of some kind or other. According to the mechanical theory of heat, radiation is a kind of wave motion, very similar in many respects to that produced when a stone is dropped into a pool of water. At the point where the stone strikes the liquid a disturbance is set up, and this disturbance spreads out on all sides, producing waves in the liquid. And, in a somewhat similar manner, the motion of the molecules of a hot body is believed to start waves in a fluid which is assumed to fill all space. This wave motion is transmitted by this substance in all directions, producing heat again when it reaches certain bodies, by imparting motion to their molecules. The substance in which these waves are believed to travel has been called the luminiferous (or light-carrying) ether; and it is assumed to fill all space, and to have a density compared with which the density of our lightest gas is very great. The existence of this substance has not been absolutely proved, neither on the other hand has its existence ever been disproved; and as it helps us to explain many things which would otherwise be difficult, if not impossible, to explain, we may continue to believe in its existence. And if we regard the radiation of heat as a wave motion in this ether, we must look upon any body which can originate the waves as a body in motion.

The motion of the molecules of a hot solid body must necessarily be somewhat after the nature of vibration; that is to say, although they are constantly in motion, they never get more than a very short distance away from their central position. Hence the molecules of a solid constantly retain the same relative positions in the mass. In the case of a gas the conditions are different; the molecules are free as regards each other, and each molecule moves with considerable velocity in a straight line, until it comes into collision, either with another molecule, or with the sides of the vessel enclosing it, when it rebounds and starts off on a fresh course. The various molecules composing a gas are moving in this manner in all directions; and the pressure which a gas exerts against any substance with which it is in contact, is believed to arise from the continual striking of the molecules of the gas against the substance. The velocity of the motion of the molecules of a body may vary, the effects produced being most intense when the velocity is greatest. Just as a bell gives out a louder note when its sides vibrate backwards and forwards through a great distance, than when the extent of vibration is smaller, so the greater the velocity of the motion of the molecules the greater is the pressure of a gas, and the more intense is the sensation of heat to the touch. All bodies, so far as we know them, possess more or less heat; the coldest body we know is not devoid of heat. Ice, cold as we think it, is warm enough to melt frozen mercury; and frozen mercury would give up heat to a body colder than itself. That is to say, there is no body whose molecules are not in a state of motion, though in some cases the velocity of the motion is much less than in others.

In some cases the effect of heat on a body is not to alter the temperature of the body, but to change its state. Thus a piece of ice in front of a fire does not get any warmer; the heat which falls upon it is entirely spent in altering the relative position of the molecules, and converting the solid into a liquid. The heat is spent in doing work in the body, and is in fact converted into potential energy, due to the separation of the molecules, and the action of cohesion on them. In the same way, when a liquid boils, a quantity of heat is spent in doing the work of overcoming the cohesion, and converting the liquid into a gas; and this is stored up as potential energy in the gas. To the heat which is thus spent in altering the state of a body without altering its temperature, the name of latent heat has been given. The potential energy of a vapour is given out in the form of heat when the vapour is condensed into a gas, and the potential energy of a liquid is similarly given out when it solidifies.

Bodies are of different temperatures or degrees of heat. When two bodies having different temperatures are placed in contact with each other, there is a tendency for heat to pass from the body of higher to the body of lower temperature; and this passage of heat from one to the other will continue until both acquire the same temperature. The passage of heat along a body from a place of higher to a place of lower temperature is called **conduction** of heat, and may be illustrated by the passage of heat along a poker, one end of which is placed in the fire. Some bodies permit this passage of heat through them to take place very readily, and are called good conductors of heat; other substances conduct heat very slowly, and are called bad conductors. Metals are all good conductors of heat; while wood, and the materials of which we make our clothes, are very bad conductors of heat. If two bodies having different temperatures are placed near each other, but not in contact, they also influence each other's temperature. Heat passes from each of them to the other, and in all directions, by **radiation**; and after a certain interval of time they will both be found to be of the same temperature. There is therefore a tendency for heat to be distributed uniformly amongst all bodies; for heat will continue to pass, by conduction and radiation, from bodies of higher to bodies of lower temperature, until all are brought to one uniform condition of temperature.

EXERCISES.

1. Mention as many different sources of heat as you can.
2. Light is able to do work in photography, and in the leaves of plants; what does that prove light to be? What relations are there between light and heat?
3. Give some examples of bodies which are good conductors of heat, and of others which are bad conductors, and mention some applications of each for their good or bad conducting powers.
4. If a piece of ice were placed in a vessel of hot water, what changes of temperature would take place, and what work would be done by the heat?
5. How is the heat of our bodies produced? Mention several ways in which our bodies lose heat. Why do we choose bad conductors of heat for clothing?

LESSON 27.

THE DISSIPATION OF ENERGY.

Our study of energy would not be complete without some reference to the principle known as the **dissipation of energy**. The subject is, however, one of some difficulty, and we can only treat of it in a very general and elementary manner.

Although it is perfectly true that the total quantity of energy in the world is invariable, we must not assume that the amount of energy available for work will always be the same as at present. On the contrary, there is reason to believe that the quantity of available energy is steadily diminishing, in consequence of its conversion into a form which is no longer available for work, and this is what is meant by the dissipation of energy. We will therefore, in the first place, consider whether there is a general tendency for the various forms of energy to become converted into one particular form; and if so, what that particular form of energy is.

We have already seen that in order to make use of potential energy of any kind,—gravitation of raised bodies, distortion of elastic bodies, &c.,—it is necessary that the energy should pass into the kinetic form; that is, should be converted into motion. In the absence of any resistance, the motion of a body will continue uniform for ever. But such a condition of things is altogether contrary to our experience; there is resistance to motion of all kinds, and at all times, so far as our experience goes. A body moving through the air experiences a resistance due to the inertia of the air, and friction against it, and part of the motion of the body is converted into heat by reason of this friction. It would appear from astronomical observations that there is a definite, though slight, resistance to the motion of the planets, and other members of the solar system; and in that case there must be a corresponding diminution in their kinetic energy. In fact, every instance of motion with which we are acquainted is accompanied by more or less friction, and therefore by more or less conversion of mechanical energy into heat. When a moving body is brought to rest by collision with some obstacle, there is a development of heat proportional to the mechanical energy destroyed. The potential energy contained in the coiled-up spring of a watch can be traced, as it is given out, first into mechanical energy of the moving spring and wheels, and ultimately into heat, by reason of friction between the moving parts themselves, and between these parts and the air. In the same way the potential energy of a raised body, or of a piece of iron near a magnet, or of two separated electrified bodies, can be traced first into motion, and then either partially or wholly into heat. The energy of chemical affinity, when employed to do work, is likewise converted into heat, as in the furnace of an engine; and the energy which currents of electricity in motion possess is sooner or later transformed into heat, the wires conveying the electricity becoming heated in proportion to the resistance which they offer to the passage of the current. It appears, therefore, from what we have now stated, that there is a general tendency for all the various forms of energy to pass into that particular form of energy called heat.

It is true that heat may be converted into motion, and into several of the various other forms of energy. But experience shows that, in practice, it is impossible to reconvert the whole of the heat produced in any case, into the form of energy from which it was transformed. For example, when a stone falls from a certain height and strikes the ground, a certain quantity of heat is developed; and according to theory the heat so produced would, if all reconverted into mechanical energy, be competent to raise the stone again to the height from which it fell. But the heat is no sooner produced than it commences to diffuse itself among surrounding bodies, and under such circumstances is not fitted to do work. In the steam engine there is a very considerable loss of efficiency, in consequence of the loss of heat by radiation, conduction, &c., into surrounding bodies. The best engine yet constructed does not convert perhaps more than one-tenth of the heat produced in its furnace into useful work. It is quite possible to convert the whole of a given quantity of motion into heat, but in no case can the whole of a given quantity of heat be converted into motion.

To make any use of heat for doing work we must have bodies at different temperatures, and apply the heat as it passes from a body at a higher to a body at a lower temperature. In the steam engine, for example, the steam in the boiler must be hotter than the steam which escapes; the heat is let down, so to speak, from a higher to a lower level in its passage through the engine. It is only the heat which is thus arrested in its course which is available for work; all the heat contained in the steam and hot gases which leave the engine is so much loss. To obtain the maximum amount of work from an engine, it is therefore necessary to have as great a difference as possible between the temperature of the boiler and that of the escaping steam, or condenser (in those engines which condense the steam). But when there is such a great difference in temperature between various parts of the engine, there is a powerful tendency for heat to pass by conduction from the hotter to the colder portions, without of course doing its share of work. Therefore, what with the large quantity of heat remaining in the escaping steam, and with the loss due to the passage of heat by conduction, &c., we cannot hope, however much we perfect our engines, to convert the whole of a given quantity of heat into its full equivalent of work.

There are then three principles which we must bear in mind, in considering the quantity of energy in the world which is available for work. They are:—

(1) The various forms of energy have all a tendency to become converted into heat.

(2) Heat tends to distribute itself uniformly amongst all bodies, so as to reduce all to the same temperature.

(3) Heat can only be employed to do work as it passes from a place of higher to a place of lower temperature.

Just as a body acted on by gravity can only do work by virtue of this action if it has space through which to fall, so heat can only do work as it passes from a higher to a lower level, or temperature. And just as a small quantity of compressed air is capable of doing work, while a whole atmosphere of air all at the ordinary pressure is incapable of work, so a small quantity of heat at a higher temperature than that of surrounding bodies is valuable as an agent for doing work, whereas a universe having one uniform temperature, however high, is useless so far as its power of doing work is concerned. And this condition of uniform temperature is that towards which our universe seems to be slowly but surely tending. If such a condition of things is ever reached, although the total quantity of energy will be the same as ever, none of it will be available for work, its utility having been gradually dissipated and lost.

EXERCISES.

1. When we speak we set the air in vibration; what becomes of the energy expended in speaking?
2. Mention an example of the conversion of electricity into heat and light, which is now of great practical importance.
3. Trace, as far as you can, what becomes of the energy of an ordinary fire?
4. In walking we expend energy, what becomes of it?

EXAMINATION QUESTIONS.

(On the subjects of Lessons 1 to 27.)

1. What do you mean by force? Name several important natural forces.
2. Give examples of a force producing motion, and of a force bringing a body to rest.
3. A stone is rolling down a hill; what forces are acting on it to keep it in motion and to stop it respectively?
4. What reasons are there for calling friction a force? Write a short account of friction.
5. Give some account of the force of gravity.
6. What do you understand by the force of chemical attraction? Give some examples of its action.
7. Why is it easier to walk down a hill than up a hill?
8. What is meant by a resistance? What resistances would gradually bring a vibrating pendulum to rest?
9. When a gun is fired, what force sends the bullet out, and what force stops it?
10. Explain the effects of the resistance of the air on moving bodies, and give some examples to illustrate your answer.
11. Write out the first law of motion, and give an illustration of it.
12. What is inertia? Give several examples of its effects.
13. Why does not a train stop instantly when the steam is shut off from the cylinder? Explain the action of the brake.
14. A man riding on horseback is liable to be thrown forwards if the horse suddenly stops. Explain this.
15. What is meant by uniform velocity? Under what circumstances will the velocity of a body be uniform?
16. What is meant by "centrifugal force"? Give some illustrations of its action.
17. What is friction? Why is it easier to push a body over a smooth floor than over a rough one?
18. Upon what conditions does the amount of friction between two bodies depend? How may friction be diminished?
19. Why is it more difficult for a horse to set a boat or cart in motion, than to keep it moving when once started?
20. What is matter? How much more matter is there in a ton of wood than in a hundred-weight of coal?
21. What is the meaning of mass? What is the unit of mass? How is the mass of a body ascertained?
22. Mention some differences between snow and water. When a given quantity of snow melts is there any change in the mass of the substance?
23. Define momentum. Of two bodies, one has a mass of 2 cwts., and moves at the rate of 5 feet per second; the other has a mass of 20 lbs. and moves at the rate of 280 yards per minute; find the ratio of the momentum of the former body to that of the latter.
24. How fast must a ball weighing 9 lbs. move so as to have a momentum equal to that of a ball weighing half a hundred-weight, and moving at the rate of 24 yards per second?
25. If the same force acts successively on two bodies for the same space of time, and produces velocities of 5 ft. and 12 ft. per second respectively, compare the masses of the two bodies.
26. If a bullet is fired horizontally from a gun, in what direction will it move through the air? Show that its actual motion is due to the simultaneous action on the bullet of two forces at right angles to each other.

27. Write out the second law of motion, and give an example to illustrate its action.
28. A boat is rowed up a river with a velocity of 3 miles an hour, the stream running down with a velocity of 2 miles an hour, with what velocity could the boat be rowed down the river?
29. How must forces of 7, 10, and 3, units respectively, be adjusted if they act at a point, and are in equilibrium?
30. What is meant by the resultant of twò or more forces? Give a diagram showing the magnitude and direction of the resultant of two forces of 4 lbs. and 7 lbs. acting on a point at right-angles to each other.
31. Three cords are tied together at a point; one of these is pulled towards the north with a force of 6 lbs., and another towards the east with a force of 8 lbs.; with what force must the third cord be pulled to keep the whole at rest?
32. Two forces acting in the same direction have a resultant of 13 lbs., and in opposite directions a resultant of 7 lbs. What magnitudes have the forces?
33. In defining a force, what three particulars respecting the force must be stated? Represent by straight lines two forces acting on a point at right-angles to each other, one having a magnitude of 30 lbs., and the other of 21 lbs., according to a scale of 6 lbs. to one inch.
34. What is a parallelogram? Write out the rule called the "parallelogram of forces."
35. If the magnitudes of two forces and the angle between their lines of direction are known, how can you find the magnitude and direction of a third force which would just balance the other two?
36. What is meant by the resolution of a force? Resolve a force of 15 lbs. into two forces acting in opposite directions to each other.
37. Write out the third law of motion, and give two illustrations of it.
38. Explain fully the kick or recoil of a gun.
39. A half-ton shot is discharged from an 81-ton gun with a velocity of 1620 feet per second. What will be the velocity with which the gun will move?
40. When a man in a boat pushes with his oar against a ship in the water, which vessel will move, and in what direction? Write down the law of motion which this illustrates.
41. If a man sitting in a train, in rapid motion towards the north, fires a bullet from a gun aimed at a tree at some distance from the side of the railway, will the bullet strike the tree? If not, on which side of the tree will it pass? On what law do you base your answer?
42. Write out in your own words the three laws of motion.
43. Explain the terms force, reaction, motion, velocity, attraction.
44. What is equilibrium? Mention some bodies in equilibrium under the action of two or more forces, pointing out in each case the forces which are acting on the bodies.
45. What is gravitation? Give some illustrations of its effects.
46. On what conditions does the magnitude of the attraction of gravitation between two bodies depend?
47. Why must the length of a pendulum required to beat seconds be increased in passing from the equator to the poles?
48. Why is it easier to lift a piece of wood than a stone of the same size?
49. The force of gravity at the surface of the sun is 27 times as great as at the surface of the earth; what would be the weight on the sun of a mass of matter which weighed 28 lbs. on the earth?
50. Explain clearly the difference between weight and mass.
51. Why have all bodies on the earth's surface weight? Why does a body weigh less at the equator than in England at the sea level?
52. What is weight? Why are some substances heavier than others?

53. What is meant by the centre of gravity of a body? Mention some of its properties.
54. How could you find the centre of gravity of a thin sheet of iron? Where would its centre of gravity be if it were (*a*) round, (*b*) square?
55. Find by a geometrical construction the centre of gravity of an equilateral triangle.
56. If you had a piece of string given you, and a rod heavier at one end than the other (*e.g.* a walking stick), how would you find the point in the length of the rod where the centre of gravity is situated?
57. Why does a man lean forward when carrying a weight on his back?
58. When is a body said to be in a position of stable equilibrium? What other varieties of equilibrium do you know?
59. Explain why a falling body is constantly increasing its velocity. Which law of motion does this illustrate?
60. Why does a large stone dropped from the top of a house fall to the ground? Why will it fall more quickly than a piece of wood having the same weight?
61. Explain why heavy and light bodies all fall at the same rate in a vacuum.
62. How long would it take a stone to fall 48 yards?
63. How long must a body fall to acquire a velocity of 160 feet per second?
64. A body is thrown vertically downwards with a velocity of 30 feet per second; how far will it fall in the first second?
65. Which body will have the greater momentum on reaching the ground; one weighing 8 lbs, and falling from a height of 64 feet, or one weighing 5 lbs. and falling from a height of 144 feet?
66. What advantage is there in having a heavy rather than a light body for the bob of a pendulum? Is the rate of vibration altered by a change in the weight of the bob?
67. How may the magnitude of a force be measured? What is meant by a force of one ton?
68. Define velocity and force. In what two ways can forces be measured?
69. What is meant by "work"? What is the unit of work?
70. Find the number of units of mechanical work expended in raising 136 cubic feet of water to a height of 20 yards. A cubic foot of water weighs 62½ lbs.
71. One person, weighing 132 lbs., carries 48 lbs. to the top of a tower 105 feet high; while another person, by means of a rope and pulley, raises 6¾ cwts. to a height of 50 feet. How much more mechanical work is done by the one than by the other?
72. Find how much work a horse does in moving a cart one mile, the cart and load together weighing 3 tons, and the resistances amounting to 150 lbs. per ton.
73. How is work measured? Define "horse-power." Of how many horse-power is an engine which lifts 90,000 lbs. to a height of 66 feet in 15 minutes?
74. How many cubic feet of water can an engine of 50 horse-power raise from a mine 40 fathoms deep in one hour, supposing it to work continuously?
75. A horse walking at the rate of 3 miles an hour draws a load of 1 cwt. up a shaft; estimate in foot pounds the work done by the horse in one minute.
76. A man can do 900,000 units of work in a working day of nine hours; at what fraction of a horse-power does he work on the average?
77. An engine is capable of raising a ton weight to the height of 66 feet 16 times in an hour. What horse-power does this indicate?
78. What is meant by the term "energy?" Give examples of bodies possessing energy.
79. What variety of energy does each of the following bodies possess:—(1) a horse; (2) a piece of coal; (3) a rapidly revolving fly-wheel; (4) a bent spring; (5) wind; (6) a stream of water?

80. What is the difference between kinetic and potential energy? State what form of energy each of the following bodies possess, and whether the energy is kinetic or potential:—oil; a bent bow; a wave; the sun.
81. What kind of energy has a vibrating pendulum at the highest and lowest parts of its swing respectively? When the pendulum, left to itself, comes to rest, what becomes of its energy?
82. What do you know of the energy of a stone thrown into the air (*a*) when at its highest point, (*b*) when it has reached the ground?
83. What is energy of position, or potential energy? Give several examples of bodies possessing this kind of energy.
84. What advantage has potential energy over kinetic energy as regards its use to man?
85. What kind of energy have moving bodies generally? On what conditions does the amount of energy in a moving body depend?
86. Which of the following two bodies has the greater capacity for work,—one weighing 5 lbs. and moving with a velocity of 20 feet per second, and the other weighing 10 lbs. and moving with a velocity of 10 feet per second?
87. What is meant by the "conservation of energy?" Illustrate your answer by reference to a stone thrown into the air.
88. Give several examples of one form of energy being converted into another.
89. Mention some instances in which mechanical energy is converted into heat.
90. Give examples of heat converted into motion.
91. "Energy may be transferred, but cannot be destroyed." Give some illustrations of this statement.
92. What do you know about the nature of heat?
93. Describe an experiment to prove that heat has no weight.
94. What is the general effect of heat on bodies? Mention any exceptions with which you are acquainted.
95. A piece of red hot iron is taken from a fire and laid on the ground; what becomes of the heat of the iron? Does all the heat pass from the iron?
96. What instrument is used for measuring the degree of heat of a body? Would the instrument indicate any difference between the heat of a pint and of a gallon of boiling water?
97. Mention several examples of the application of heat to perform useful work.
98. What sources of heat would be available to us in the absence of wood and coal?
99. Whence is the energy of the following bodies derived:—a man; coal; a waterfall; wind.
100. Why is a mass of stone elevated above the ground able to do work, while the same mass lying on the ground has not that power?

LESSON 28.

MACHINES.

A machine is an instrument by means of which a force may be applied in such a manner as to produce a certain definite effect. The instrument itself cannot originate the force, but moves and works only by virtue of external force applied to it. Its function is merely to transmit the applied force to the required place, and to apply it in the required manner. By means of a machine energy may be transferred from one body to another (*e.g.*, from the wind to the mill-stones of a windmill), and applied in doing work of a particular kind. But the machine has no power to create energy, or to change the amount of energy supplied to it. It usually happens, however, that the machine spends in useful work only a portion (often a very small portion) of the energy supplied to it, the rest of the energy being expended in overcoming the internal resistances (friction, &c.) of the machine itself.

A machine may consist of solid parts only, or of solids combined with liquids or gases. Solids, liquids, and gases are all capable of transmitting force, and may therefore be employed in the construction of machines.

Any force may be employed to drive a machine, provided that the machine is so constructed as to enable the force properly to act upon it. Thus, in order to utilise the force of the wind, the machine must have connected with it sails (as in the case of a ship), or arms (as in the case of a windmill), spread out so as to be acted on by the pressure of the wind. In order to utilise the force of a stream, a wheel having pieces of board projecting from its rim may be set up in the stream, so as to be set in motion by the moving water. Or, if the water be dammed up to form an elevated reservoir, a water-wheel of somewhat different construction may be employed, having a number of compartments or buckets arranged round its circumference. The water being led from the reservoir to the uppermost of these buckets fills them, and they by their weight force down that part of the wheel, thus bringing others into position to be filled, while each empties itself as it passes the lowest part of its path. If, as is the case with steam engines, it is the energy of chemical affinity to be employed, the machine must be constructed with a fire-place so arranged as to be supplied with a constant draught of air, and combined with a boiler in which steam may be formed; while there must be a cylinder with a piston, or some other arrangement, on which the steam can act. And so for all the other forces, the machine must be adapted in each case to the particular force which it is desired to employ.

In a previous lesson we have explained that there are three elements to be considered with regard to a particular force, viz., its point of application, its direction, and its magnitude. By means of a machine, one or more of these elements may be modified. Thus, by pulling at the rope of a pulley (fig. 13, *P*) with a certain small force in a downward direction, the force may be transmitted by the rope round the various wheels, and applied to raise a very heavy weight (*W*). In this case the point of application, direction, and magnitude of the force exerted on the rope, have all been changed by means of the machine, *i.e.*, the pulley. If we compare with this the case of a man standing on a scaffold and drawing up a number of bricks by means of a rope attached to them, we shall see that the rope transmits the force from the man's hand to the weight, and therefore alters the point of application as in the former case. But the direction and magnitude of the force transmitted to the bricks are (if we neglect the weight of the rope itself) exactly the same as the direction and magnitude of the force applied by the man to the rope. On this account we should not consider the rope as a machine, but limit the term **machines to instruments which transmit forces applied to them, and which during transmission modify the direction or magnitude of the forces.**

It is in this power of modifying a force in certain respects, as well as of transmitting it, that the utility of a machine depends. A force acting in one direction may, by the interposition of a suitable machine, be made to impart motion to a body in an entirely different direction. A force acting in a straight line may be made by means of a machine to maintain the revolution of a wheel. (This is illustrated by fig. 12, where the rectilinear motion of the piston rod *P*, is converted into the circular motion of the wheel by means of the crank *C*, and connecting rod *R*.) In the same way a small force acting on a suitable machine may be sufficient to lift a body, which, if applied directly, it would be unable to move. A small force applied to the pulley represented in fig. 13 at *P*, would be converted by the machine into a much larger force acting at *W*. By a different arrangement, again, a body may be made to move with a very much greater velocity than that of the part of the machine to which the force is applied.

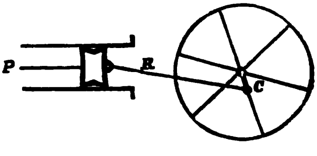

Fig. 12.

But the question might be asked, how does the fact that a machine can convert a small force into a larger one, and a force acting through a short distance into one acting over a much greater distance, agree with the principle of the conservation of energy, and with the statement above that a machine cannot increase the quantity of energy supplied to it? The answer is fortunately neither obscure nor difficult. It has long been known that when a machine has been used to gain power, that is, when a small force applied to the machine is converted by it into a larger force, there is a loss in speed; the weight moved by the machine moving more slowly than the portion of the machine to which the power is applied. In the pulley, fig. 13, for instance, the weight *W* would rise much more slowly than the power *P* would fall. A force of about 56 lbs. applied at *P* would suffice to raise a weight of 3½ cwts. hung at *W* (supposing the pulley to work without friction). But the power *P* would move 7 times as fast as the weight *W* would rise; in other words, to raise *W* one inch, *P* must move down through 7 inches. While the machine, therefore, increases the magnitude of the force seven-fold, the velocity with which the body *W* moves, and the distance through which it travels, are diminished in the same proportion. This is the case, not only with the pulley represented, but with every machine, and the principle is usually stated as follows:—**What is gained in power, is lost in speed.** If a force were applied to the pulley at *W* to raise a weight hung from *P*, then the latter body would move 7 times as fast as the power; but a force of 7 lbs. at *W* would be required for every 1 lb. hung at *P*. So that in this case what is gained in speed is lost in power; and the same is true for every machine. By no machine, or combination of machines, would it ever be possible to gain both in power and in speed at the same time; a gain in the one respect is inevitably accompanied by a corresponding loss in the other. This was pointed out by Newton in connection with his laws of motion; "the power and use of machines consist only in this, that by diminishing the velocity we may increase the force, and the contrary."

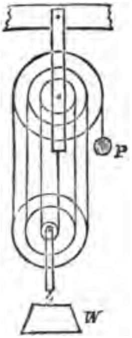

Fig. 13.

The principle which we have just stated, and which is known as the "principle of work," is most important in connection with the theory of machines. In considering the conditions under which a machine works, we have to consider the relation between two forces,—one, called the **power**, which is applied to the machine to move it, and another, called the **resistance** (or sometimes the **weight**), which tends to prevent motion, and which has to be overcome by the machine. The power expends energy or does work *on* the machine, and a certain amount of work is done *by* the machine in overcoming the resistance. The principle of work to which we have referred asserts that, in the most perfect machine (where no energy is wasted in overcoming resistances amongst the parts of the machine itself), the work done by the power is equal to the work done against the resistance. Or, to state it in another form, the energy expended by the power on the machine is exactly equal to the energy expended by the machine in overcoming the resistance. In an ordinary machine energy is wasted during transmission, being converted into heat by the friction of the parts; but the energy so wasted, together with that expended by the machine in doing work, is equal to the amount of energy applied to the machine. In the case of the pulley in fig. 13, which we assume to work without friction or other resistance, a force of 1 lb. applied at *P*, and moving its point of application 7 feet, would do 7 units of work. As we have explained above, this force transmitted by the cord over the pulley would suffice to raise a weight of 7 lbs. through the space of one foot, which is again equal to 7 units of work. The principle of the conservation of energy, therefore, holds good in the case of machines.

It is found possible to regard all machines as merely combinations of a few simple arrangements, which are known as **simple machines** or **mechanical powers.** Of these six are usually enumerated, as follows:—(1) the **lever**; (2) the **wheel and axle**; (3) the **pulley**; (4) the **inclined plane**; (5) the **wedge**; (6) the **screw**. The theory and applications of these mechanical powers will form the subject of the succeeding lessons. The main problem in each case will be to determine the conditions of equilibrium, or in other words, the relation being the power and the resistance when those forces balance each other.

EXERCISES.

1. Mention several compound machines, stating in each case the nature of the power applied, and of the resistance overcome.
2. Show that a pair of ordinary weighing scales satisfies our definition of a machine. What relation is there between the power and resistance when the scales are balanced?
3. Describe some arrangement by means of which a force applied to turn a handle round and round can be made to raise a weight vertically, *e.g.*, a bucket of water from a well.
4. A machine is arranged so that a force of 42 lbs. applied to it can raise a weight of half a ton; compare the velocity of the weight with that of the power.
5. Mention some precautions which may be adopted in order that a machine may give out as large a proportion of the energy spent on it as possible.
6. Mention some instances in which a force is transmitted by a liquid and a gas respectively.

LESSON 29.

THE LEVER.

A lever is a rigid bar capable of turning freely about a fixed point or axis. This fixed axis is called the **fulcrum** of the lever, and may be situated at any part of the bar. The distances, measured in a straight line, from the fulcrum to the point of application of the power and of the resistance respectively, are called the **arms** of the lever. Every lever has two arms, which may be either equal to each other or unequal. The term rigid used in the definition means stiff or inflexible; that is, the bar must not give way or bend under the action of the forces applied to the lever. No substance known, however, is perfectly rigid; even steel, which is one of the most rigid substances, yields slightly under the action of comparatively small forces. But there are many substances which are sufficiently rigid to form levers for all practical purposes. A bar of whale-bone or cane, which would bend if applied as a lever, would form a very unsatisfactory lever, since much of the power applied would be spent in bending the bar, and not in raising the weight. But a stout wooden bar or an iron rod bends so little under ordinary circumstances, that we may consider it sufficiently rigid to satisfy our definition of a lever. It is not necessary that the lever should be strictly in the form of a bar; a body of any shape may be regarded as a lever, provided that one part of it is supported so as to form an axis or fulcrum, and that the rest of the body is able to move freely about this axis. A box lid, for example, may be regarded as a lever, the fulcrum being at the hinges. Neither is it necessary that the lever should be straight: there are bent levers as well as straight ones; and the only essential condition required to constitute a body a lever, is that it is capable of turning about an axis. The conditions of equilibrium in the case of a bent lever are more difficult to understand than in the case of a straight lever, and we shall chiefly concern ourselves with this latter variety. By a straight lever we mean one in which the two arms are in the same straight line.

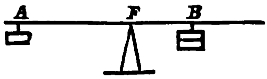

Fig. 14.

The lever has been practically employed from very early times; and many portions of our own bodies, and those of animals generally, are arranged so as to act as levers. The construction of the lever is so simple, and its efficiency so great, consisting as it does of only one part, and being subject to friction at only one point—the fulcrum—that we cannot wonder at it being so very commonly employed. More than 2,000 years ago Archimedes, a famous philosopher, discovered the rule or principle of the lever. He showed that if a bar rested on a fulcrum (fig. 14, *F*), and was loaded with two heavy bodies having unequal weights, there would be equilibrium when the length of the arm on which the lighter body hung bore the same relation to that of the arm on which the heavier body hung, as the weight of the heavier body did to that of the lighter. Thus in fig. 14 the lever loaded with the two bodies *A* and *B*, of which *B* has twice the weight of *A*, will be in equilibrium when the distance *FA* is twice as great as the distance *FB*. If we call the arm at which the power acts the power-arm, and the other the resistance-arm, the rule for equilibrium in a lever may be stated thus:—**the power must be to the resistance in the same proportion as the resistance arm is to the power-arm**; or, in another and very convenient form,

$$\frac{\text{P}}{\text{R}} = \frac{\text{resistance arm.}}{\text{power arm.}}$$

This is equivalent to saying that, when the lever is in equilibrium, the product of the power into the length of the arm at which it acts is equal to the product of the resistance into the length of the arm at which it acts. From this it follows that when the arms are equal, the power is equal to the resistance; and this is the principle of the common weighing scales.

It is usual to distinguish three kinds or **orders** of levers, according to the relative position of the points where the fulcrum, power, and resistance respectively act. The difference, however, is merely a matter of convenience, and not one of principle. When the fulcrum is situated anywhere between the point of application of the power, and the point where the resistance acts (fig. 15, *A*), the lever is said to be of the first order. In a lever of the second order the resistance is situated between the fulcrum and the power (fig. 15, *B*). In a lever of the third order (fig. 15, *C*) the power is situated between the fulcrum and the resistance.

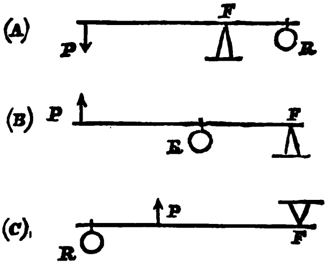

Fig. 15.

The fulcrum which supports the lever is subject to more or less pressure, the amount depending upon the magnitude of the forces, and the directions in which they act. The fulcrum reacts on the lever with a force equal to the pressure on it, and when the lever is in equilibrium, it is because the three forces acting on it—the power, resistance, and reaction of the fulcrum—are so arranged that the effect of any one of them will just counteract the combined effect of the other two. In a lever of the first order the power and resistance act in the same direction, and the pressure on the fulcrum (and therefore the reaction of the fulcrum) is equal to the sum of the power and resistance. In the second and third orders the power and resistance act in opposite directions, and the pressure on the fulcrum is equal to the difference between these forces. In the second order the resistance, acting on the shorter arm, is greater than the power, and is in fact equal to the reaction of the fulcrum together with the power. In the third order the power has the shorter arm, and therefore the greater magnitude, being equal to the resistance together with the reaction of the fulcrum.

So far we have said nothing about the effect of the weight of the lever itself. In theory the lever is assumed to be without weight, and the principle of the lever, as stated above, is based on that assumption. But in practice the weight of the lever has to be taken into consideration. When the lever is of the same material, and of uniform thickness and density throughout, and the fulcrum is placed in the middle of the bar (as in fig. 14), it is evident that one half of the bar will balance the other half. In this case the principle as stated will hold good, and the only effect of the weight of the lever will be to increase the pressure on the fulcrum. But when the centre of gravity of the lever is not at the fulcrum, the weight of the lever must be considered as an additional force acting at its centre of gravity; in some positions this will increase, in others diminish, the resistance to be overcome by the power in moving the lever.

The principle of work applies to the lever as well as to the other mechanical powers,—viz., that in moving from one position of equilibrium to another, the work done by the power is equal to the work done against the resistance. Thus, in fig. 16, PRF is a lever of the second order; the power arm PF is twice as long as the resistance arm RF, and therefore $R=2P$. When the lever is moved by the power into the new position $P'R'F$, the point of application of the power travels through a distance PP', which can be proved by geometry, or by actual measurement, to be twice as great as the distance RR', through which the point of application of the resistance is moved. Then, multiplying the force by the distance through which its point of application is moved, to find the amount of work done, it is obvious that $P \times PP' = R \times RR'$. The same is true for every lever, the distances through which the points of application of the forces are moved being inversely proportional to the magnitudes of the forces. By means of a lever it is possible to gain at will either power or speed. To gain power the force must be applied to the longer arm, while to gain speed the force must act on the shorter arm.

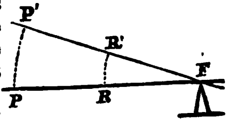

Fig. 16.

If the lever is not straight, or if the Power and Resistance do not act in directions parallel to each other, there will be equilibrium when the forces are inversely proportional to the lengths of the perpendiculars drawn from the fulcrum to the lines of direction of the forces. Thus, in the case of the bent lever represented in fig. 17, the forces P and R will keep the bar in equilibrium if $P : R$ as $Fr : Fp.$, or if $P \times$ distance $Fp = R \times$ distance Fr.

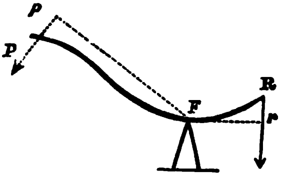

Fig. 17.

EXERCISES.

1. In the first kind of lever the power arm is 8 inches long, the resistance arm 3 inches long, the power 15 lbs.; find the resistance.
2. At what distances from the fixed point will weights of 10 lbs. and 2 lbs. balance each other on a straight lever?
3. The arms of a lever are 9 inches and 25 inches long respectively; what weight on the shorter arm would balance 45 lbs. on the longer arm? If the lever is of the first order, what will be the pressure on the fulcrum?
4. What force acting 4 feet from the fulcrum will balance a force of 80 lbs. acting at a distance of 15 inches?
5. Weights of 1 lb. and 10 ozs. balance each other on a lever; the long arm is 3 feet long, find the length of the short arm.

6. A bar 6 feet long has attached to it at one end a weight of 20 lbs. and at the other 16 lbs.; at what point in the bar would it balance on a fulcrum?

7. A bar 44 inches long is in equilibrium with weights of 5 and 6 lbs. respectively hung from its ends. Find the position of the fulcrum.

8. Two weights together equal to 9 lbs. are hung on a lever in equilibrium, one is 5 times as far from the fulcrum as the other is. What are the weights?

9. A straight bar 12 feet long is arranged as a lever of the first order, having its fulcrum at a distance of 18 inches from one end; what force on the long arm will support a weight of 2 cwt. on the short arm.

10. The arms of a lever are in the proportion of 3 to 5; what power applied to the short arm will support a weight of 24 lbs. on the long arm? Draw diagrams representing the lever as of (*a*) the first, (*b*) the third order.

11. A lever of the first order has one arm 3 inches long, and the other 4 ft. 9 in. long; find what power will be required to overcome a resistance of 380 lbs. What distance must the power move in order to move the resistance 1 inch?

12. The pressure on the fulcrum of a lever of the first kind is 36 lbs., and the arms are respectively 20 inches and 10 inches long; find the power and the resistance.

13. The arms of a lever are respectively 20 and 21 inches long; find what force on the long arm will balance 40 lbs. on the short arm; and what force on the short arm will balance 40 lbs. on the long arm.

14. A bar 10 feet long is hinged at one end, and a weight of 6 lbs. is hung on the other end and balanced by a force acting at 15 inches from the fulcrum. Draw a diagram to illustrate this arrangement, showing the direction in which the force acts, and find the magnitude of the force. Find also the pressure on the hinge.

15. Two forces of 15 and 20 lbs. respectively balance each other on a lever of the second order, the shorter arm of which is 9 inches long; find the length of the lever.

16. A screw-driver 18 inches long is inserted $\frac{1}{2}$ inch between two boards, and used as a lever to force the boards asunder. If the force applied to the screw-driver be 12 lbs., what will be the pressure (*a*) on the board against which the end of the screw-driver presses, (*b*) on the edge of the board acting as fulcrum?

17. Draw a diagram of each of the three orders of levers representing a force of 20 lbs. as supporting a weight of 100 lbs., and state in each case the pressure on the fulcrum.

18. A weight of 11 lbs. is hung on a lever, and is supported by means of a downward pressure of 7 lbs.; what order of lever is employed, and what is the pressure on the fulcrum?

19. It is required to arrange a lever so that the resistance shall be moved 1 foot when the power moves 1 inch; show by a diagram how it may be done.

20. A bar of uniform thickness 3 feet long weighs 4 lbs., and is used as a lever of the second order to support a weight of 10 lbs. hung at a distance of 1 foot from the fulcrum. Find the power required.

LESSON 30.

APPLICATIONS OF THE LEVER.

A lever may be employed to gain either power or speed, and examples of the application of the principle of the lever for one or other of these purposes are very numerous. A lever of the first order may be used to gain either power or speed, according as the force is applied to the longer or shorter arm. Levers of the second order, where the power and resistance act both on the same side of the fulcrum, and where the power is always farther from the fulcrum than the resistance is, must necessarily increase the power at the expense of the speed; whereas in levers of the third order the reverse is the case.

The see-saw, which children often arrange by resting a plank on some support, is a lever of the first order. If the children riding are of equal weight they must be at equal distances from the fulcrum; but if they are unequal in weight the lighter one must be at the greater distance from the fulcrum, and will move through the greater space. The crowbar, used in the manner shown in fig. 18 to raise a heavy body, is a lever of the first order, the weight of the heavy body on the shorter arm constituting the resistance to be overcome. Other examples of applied levers of the same order are to be found, in the poker rested at one point on the bar and employed to raise up the coals, in the handle attached to the rudder of a barge, and in the handle of a pump. Scissors and pincers are examples of double levers of the first order working on a common fulcrum. The common balance and steelyard also belong to this order of levers, but on account of their importance we shall devote a special lesson to them.

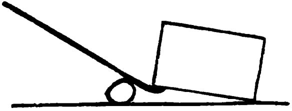
Fig. 18.

The crowbar may be used as a lever of the second order, as in fig. 19, the ground serving as the fulcrum for the end of the bar, while the resistance acts at a point somewhat above the fulcrum. The ordinary nut-crackers and lemon-squeezers are double levers belonging to this order. The knife pivoted at one end, which is used for cutting loaf sugar, turnips, &c. and the ordinary wheel-barrow, act as levers of the second kind. In the case of the small flour-truck, and of the porter's barrow used at the railway stations, they may be used as levers of the second order; but when sufficiently tilted the weight

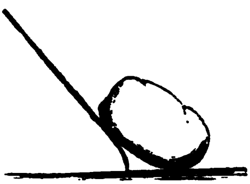
Fig. 19.

may act partly on one side of the fulcrum, and partly on the other, so as to convert them more or less into levers of the first order. In rowing a boat the oar acts as a lever of this kind, the fulcrum being at the place where the oar rests against the water, while the pressure of the oar on the row-lock serves to overcome the resistance which the boat encounters in moving through the water.

Levers of the third order are usually employed where the object is to gain in speed. One of the simplest arrangements of this kind is to be found in the treadle connected with lathes, knife-grinding machines, &c. One end of the treadle rests on the ground, or on some part of the machine, which thus forms the fulcrum, while the other end is connected by means of a connecting rod and crank with the wheel to be moved. The foot being placed somewhere about the middle of the treadle, a considerable velocity can be maintained without the foot moving inconveniently fast or far. The fire-tongs consist of two levers of the third order working together; and when the hand is applied near the hinge, as it usually is, a slight movement of the fingers opens the end of the tongs to a much greater distance. There is of course a considerable loss of power in the tongs, so that it would be impossible to crack a nut with them, as ordinarily used; but in spite of the loss there is still sufficient force at the ends to prevent the coal falling out. The sugar tongs and spring shears are constructed on this principle. The steam in raising the safety valve of the boiler of a steam engine has often to act on a lever of the third order, as in fig. 21, where *V* is the valve connected with the lever, which has its fulcrum at *F*, and is loaded by a weight *W*. Accidents have sometimes happened in consequence of the safety valve being overloaded, by an extra weight added at *W*, probably by persons ignorant of the principle of the lever, and the manner in which a small force applied at *W* would be equivalent to a much larger one at *V*.

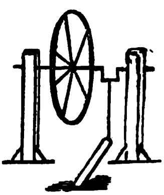

Fig. 20.

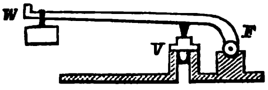

Fig. 21.

Examples of all the different orders of levers may be found in the human body. Thus, an important example of a lever of the third order is found in the bones of the fore-arm jointed at the elbow (fig. 22, *F*). The power which is furnished by the contraction of the biceps muscle—a mass of flesh which can be felt in front of the upper arm (*B*)—acts just in front of the elbow-joint (at *P*), while the weight of the hand and fore-arm, together with that of any body held in the hand, which constitutes the resistance, acts at a much greater distance. There is, therefore, a great loss in power, but gain in speed. After the arm has been bent by the action of the

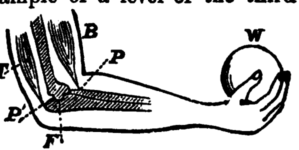

Fig. 22.

biceps muscle, it may be straightened again quickly, as in striking a blow, by means of another muscle behind the arm, known as the triceps (fig. 22, *T*), which acts on the bone of the fore-arm just behind the elbow-joint. This, therefore, constitutes a lever of the first order, where the power is applied to the shorter arm. In raising the body on tip-toe, the large muscle of the calf of the leg acts on the heel bone at *P* (fig. 23), while the weight of the body is transmitted by the leg-bone to the ankle at *W*, the fulcrum being where the toes rest on the ground. This arrangement, therefore, constitutes a lever of the second order.

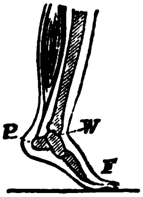

Fig. 23.

The ordinary bell-crank is an example of the application of a bent lever; and in this particular case, where the arms are equal, its only use is to change the direction of the force.

Problems connected with the distribution of the weight of a loaded bar on its supports may readily be solved on the principle of the lever. Thus, in fig. 24, if the weight *W* were hung in the middle of the bar, the pressure produced by it would act equally on each support, the pressure on *A*, for example, being equal to $\frac{W}{2}$. When the weight does not hang in the centre, we may find the proportion of it borne by one support, for example *A*, by considering the bar as a lever of the second order, the reaction of *A* being the power, and *B* the fulcrum. By treating it in this way, it may be proved that the pressure on *A* is to the pressure on *B*, as the distance *BW* is to the distance *AW*. Thus, in fig. 24, *BW* is 4 times as long as *AW*, and the support *A* will bear four-fifths, and *B* one-fifth, of the weight of *W*.

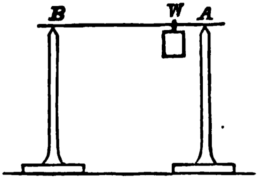

Fig. 24.

EXERCISES.

1. Why is the axle of a cart placed under the centre rather than at one end? How should the load be arranged so that the horse may have to bear as little of it as possible?
2. What order of lever is formed by the lid of a box? Why is a person's finger more severely hurt if shut in near the hinge, than if caught nearer the front of the box?
3. Why is it easier to shut a door with the hand near the outer edge, than with the hand near the hinges?
4. Why is a piece of wire to be cut with scissors placed near the rivet?
5. The handle of a hammer 15 inches long is acted on by a force of 48 lbs., in order to draw out a nail by the claw of the hammer, at a distance of 3 inches from the point acting as fulcrum. Find the magnitude of the force exerted on the nail.
6. A man raises a piece of timber by a bar 6 feet long, the fulcrum being a stone at a distance of 6 inches from the end of the bar. The pressure exerted by the man on the bar is 112 lbs.; find the pressure on the timber, and on the stone.

7. Two boys wish to play at see-saw on a plank 15 feet long, one boy weighing 80 lbs. the other 70 lbs.; how will they arrange the plank? Which boy will have the better ride?
8. The distance from the axle of the wheel of a wheel-barrow to the end of the handles is 4 ft., and from the axle to the centre of gravity of the barrow and its load 8 inches. If the weight of the barrow and load together is 1½ cwt., find the amount supported by the man and the axle respectively.
9. Two men move a boat with oars 7 ft. long, the distance from the hand to the row-lock being 2 ft. Each man pulls on the oar with a force of 36 lbs.; find the total force exerted on the boat.
10. A rod 12 inches long rests on a point at each end, and a weight of 10 lbs. is hung at a distance of 4 inches from one end; find the pressure on each point.
11. A horizontal bar weighing 112 lbs. rests by its ends on two supports; what is the pressure on each support? An additional weight of 112 lbs. is then hung at a distance of one-third the length of the bar from one support, find the total pressure on each support.
12. A man raises a ladder 30 feet long by one end, the other end resting on the ground; what order of lever does this illustrate? Assuming the weight of the ladder to be 96 lbs., and the centre of gravity 16 ft. from the end being lifted, find the pressure on the man's hand and the ground respectively.

LESSON 31.

THE BALANCE AND STEELYARD.

The common weighing balance is a lever of the first order, in which the arms are equal. Equal masses hung on the arms at equal distances from the fulcrum balance each other; and the instrument is used to determine whether two given masses are equal, one of the masses being usually a standard weight. The movable part of the balance from which the weights are suspended, and which in fact constitutes the lever, is called the **beam** (fig. 25, *A B*). The beam should be so constructed as to be as rigid as possible, and not liable to have its form altered by the action of the weights which it will have to support; while at the same time it should be as long and as light as possible. The fulcrum of a good balance is composed of a triangular piece of very hard steel, technically known as a "knife edge," and the fine edge of this rests upon either a hard polished plate of steel or of agate; agate being a very hard variety of stone which takes a very smooth and durable surface. In order that the lengths of the arms may be sharply defined and constant, a good balance has the pans also suspended from knife edges. In this case the lengths of the arms will be measured from the knife

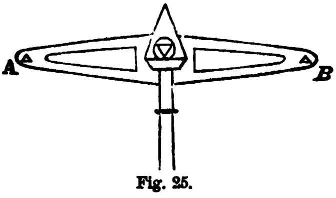

Fig. 25.

edge on which the beam rests, to the knife edges on which the pans hang. As a rule, the beam is so arranged that the straight line joining the points of suspension of the pans, passes through the point of suspension of the beam.

The beam should be suspended so as to rest in a position of stable equilibrium, in order that it may tend to return to its original position of equilibrium after it has been displaced. This condition, which is obviously essential to a good balance, is secured by having the centre of gravity of the beam below the point of support. The sensibility of the balance, however, is increased by having the centre of gravity very near the point of support. The arms of the balance should be of exactly the same length, and the pans hung from them should be of exactly the same weight.

In examining a balance as to its accuracy, it is not sufficient to observe that the beam with the unloaded pans suspended from it stands horizontally. It might be that one arm was slightly longer than the other, and compensated by having a somewhat heavier pan hung on it; but such a balance would not weigh correctly. In such a case, if the pans where changed to the opposite sides, the equilibrium would be destroyed, whereas there would be no difference if the balance were correctly constructed. Or a weight might be placed in one scale, and in the other some substance to balance it; then if each body were removed and placed in the opposite pan, the equilibrium ought to be maintained.

It is quite possible, however, to weigh accurately with an inaccurate balance, by the method known as "double weighing." Suppose that it were required to weigh out accurately a pound of mercury by means of a balance known to be inaccurate. A pound weight would be placed in one pan, and in the other sufficient sand, or shot, or other substance, to balance the pound weight. Then if the pound weight were removed, and mercury put in its place until it just balanced the substance in the other pan, it is evident that, however inaccurate the scales might be, the weight of the quantity of mercury is exactly the same as that of the pound weight employed. In cases where great accuracy is required, this method of double weighing is employed even with the best balances, so as to correct the almost inevitable slight inaccuracies of the balance.

When the balance is made very sensitive, or delicate, the beam oscillates many times before coming to rest; and much time would be lost if it were necessary to wait until it did come to rest, before it could be seen whether the weights in the two pans were equal. To avoid this loss of time it is usual to attach a long pointer to the beam, which swings backwards and forwards in front of a graduated scale. When the pans are equally loaded, the pointer as it swings makes an excursion of practically the same length on each side of its position of rest, whereas, when the pans are unequally loaded, the excursion is greater on the side of the smaller load. By this means the equality or non-equality of the weights is seen at once, and much time is thus saved.

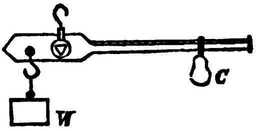

Fig. 26.

The **steelyard** is another example of a lever applied to determine the weights of bodies. It is a lever of the first order, made of steel (on account of its great rigidity), and resting by means of knife edges on rings attached to the hook by which it is held or suspended (fig. 26). The arms are very unequal in length, the substance to be weighed (*W*) being hung from the shorter arm, and counterpoised by a movable weight (*C*) on the longer arm. The longer arm of the steelyard is graduated, showing the positions at which the counterpoise must be placed to balance bodies of different weights hung at *W*. The graduation may be made as the result of calculation, or of experiment. In the latter case weights of 1 lb., 2 lbs., 3 lbs. &c., are hung at *W*, and the point on the longer arm where the counterpoise produces equilibrium is found and marked. The graduations are at equal distances from each other; that is, the distance from the mark on the arm indicating a weight of 1 lb. to that indicating 2 lbs., is equal to the distance between 2 and 3, 3 and 4, &c. Some steelyards have on two opposite sides two distinct graduations, one being intended for use with very much heavier bodies than the other. The same counterpoise is used for both graduated scales; but when the heavier bodies are to be weighed the steelyard is suspended by another hook, nearer to that from which the body to be weighed is hung, and the latter therefore acts on a shorter arm than before.

The variety of steelyard we have described is that which is known as the Roman steelyard, and is the one most generally used. There is, however, another variety known as the Danish steelyard, where the counterpoise is fixed at the end of the bar, and the fulcrum is movable.

EXERCISES.

1. A balance has its arms 10 inches and 11 inches long respectively; find the apparent weight of a body, whose true weight is 110 ozs. when hung (*a*) from the short arm, (*b*) from the long arm.
2. A body whose true weight is 1 lb. appears to weigh $15\frac{3}{4}$ ozs. in the scale pan of a balance, find the relative lengths of the arms of the balance.
3. If the arms of a balance were slightly unequal in length, how would you find out, without actually measuring, which was the longer arm?
4. What precautions would you take to keep a balance and a steelyard in good order?

LESSON 32.

THE WHEEL AND AXLE.

When a considerable force has to be exerted through only a small distance, an ordinary lever is a very suitable instrument to employ. But where the resistance has to be overcome through a great distance (*e.g.* in raising a bucket of water from a well), the lever in its ordinary form cannot be used. For, suppose a lever with arms 54 inches and 6 inches long respectively were used, it would give a mechanical advantage of 9 to 1; but while the resistance was lifted 1 foot the power would move 9 feet. With such a lever it would be impossible to raise a weight more than a short distance, without the necessity of raising the fulcrum several times. This difficulty is overcome by the employment of a special form of lever, or rather of a series of levers, under the name of the **wheel and axle.** This mechanical power is met with in various forms, but in its simplest form it is represented in fig. 27. It consists of two wheels or cylinders, having different diameters, but both fixed so as to move together on a common axis or fulcrum *F*. The larger cylinder or wheel is called the **wheel**, the smaller is called the **axle**; and round each a cord passes (but in different directions) to which the forces are applied. The mechanical advantage of the arrangement depends on the fact that a small force *P*, acting on the wheel, will balance a larger force *W*, acting on the axle. When *P* is pulled down so as to uncoil the rope on the wheel, the other rope is coiled on the axle, and the weight *W* is raised; and this will go on continuously as far as the ropes will allow. Thus it is possible, by means of this arrangement, to lift a weight through a very much greater distance than could be conveniently done by means of a lever of the ordinary kind. The wheel and axle is a lever, as we shall now proceed to show, but it is a lever arranged to work continuously.

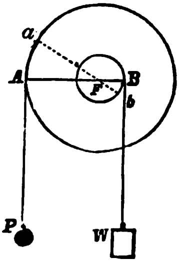

Fig. 27.

A lever may have any shape whatever, and we may regard any body as a lever, provided it is capable of being turned about a fixed point or fulcrum. The pivot which supports the wheel and axle in fig. 27 is the fulcrum, and the weights *P* and *W* may be considered as acting on the arms of a lever *AB*, and tending to turn the lever in opposite directions. All the wheel and axle might be cut away except the bar *AB*, and (if we neglect the weight of this bar) equilibrium would still be maintained. So long as the whole is at rest, we have, therefore, only to consider the lever *AB* as supporting the weights. When the wheel and axle is turned, the portion *AB* passes on, and another similar lever (such as *ab*) comes to take its place, and this will go on continuously. We may, therefore, look upon the wheel and axle as being composed of a number of levers, arranged side by side in such a manner that they come into action one after another, as the apparatus revolves. The longer arms of the lever form the wheel, and the shorter arms the axle; and this not only proves that the principle of the wheel and axle is the same as that of the lever, but leads us directly to the conditions of equilibrium in the wheel and axle. The longer arm of the lever is obviously equal to the radius of the wheel, and the shorter arm to the radius of the axle; so that the power is to the resistance as the radius of the axle is to the radius of the wheel; or

$$\frac{P}{R} = \frac{\text{radius of axle}}{\text{radius of wheel.}}$$

From this it also follows that P is to R as the diameter of the axle is to the diameter of the wheel, or as the circumference of the axle to the circumference of the wheel.

The principle of work is true in the case of the wheel and axle, a gain of power being attended with a loss of speed (or space), and *vice versâ*. When wheel and axle (fig. 27) have made one complete revolution, the power will have fallen through a distance equal to the circumference of the wheel, and the weight will have been raised through a distance equal to the circumference of the axle. If the radius of the wheel is 5 times that of the axle, the circumference of the wheel will likewise be 5 times that of the axle, and P will be equal to $\frac{W}{5}$. Then, since P is equal to $\frac{1}{5}$ of W, and the distance through which P moves is 5 times as great as that through which W moves, it follows that the work done by P is equal to the work done on W; or the work done by the power is equal to the work done against the resistance. The wheel and axle may be used either for gaining power or speed; in the former case the power is applied to the wheel, but when it is desired to gain velocity the power is applied to the axle.

The wheel and axle represented in fig. 27 is evidently a lever of the first order, and the pressure on the fulcrum is equal to $P + W$, together with the weight of the ropes, and of the wheel and axle, if we take those into consideration. In some cases the power and resistance act in opposite directions on the same side of the fulcrum, constituting a lever of the second order; and the pressure on the fulcrum is then equal to the difference between the power and the resistance.

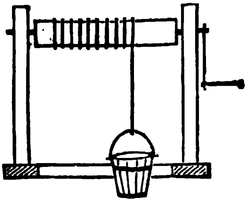

Fig. 28.

The **windlass** (fig. 28) is one of the most familiar practical applications of the wheel and axle, and is employed to raise water from wells, stones and soil from excavations, and for many other purposes. A portion only of the wheel is present in the form of a handle, but that makes no difference whatever in the principle. The barrel on which the rope or chain coils is the axle; and the ratio of the power to the resistance is the same as that of the radius of the barrel to the length of the handle—the length of the handle obviously being the radius of the wheel.

The **capstan** used on board ship for raising the anchor, &c., is a very similar arrangement, but the cylinder or barrel on which the rope coils is vertical, and is worked by means of several horizontal handspikes (like the spokes of a wheel), thus admitting of several men acting on it at the same time.

The **crank and wheel** represented in fig. 12, p. 84, and fig. 20, p. 92, and employed to convert rectilinear into circular motion in the case of a steam engine, sewing machines, &c., is another example of the application of the wheel and axle. The crank represents the axle, and as the power is applied to the crank there is a gain in speed. In the case of a bicycle, the great difference between the radius of the wheel and the arm of the crank on which the foot works gives a very considerable increase of speed, accompanied, of course, by a corresponding loss of power.

Wheels are sometimes intended to act upon each without being fixed on the same axle, and they are then connected by means of belts, or by the friction between the circumference of the one wheel and that of the other. To prevent slipping, it is usual to cut teeth upon the circumferences of the wheels, which teeth catch into each other as the wheels revolve (fig. 42). The circumferences of two wheels thus connected have the same velocity, and the smaller wheel will make a greater number of revolutions than the larger wheel, in the same proportion as the circumference of the larger is greater than that of the smaller. If, then, on the same shaft as that which carries the smaller of the two wheels, a large wheel is connected, it will revolve as frequently as the smaller wheel, and its circumference will therefore move with a greater velocity. In this manner the force is transmitted from the first large wheel, by a wheel and axle arrangement, to the second large wheel, and velocity is gained. If, on the other hand, the driving wheel were connected with the larger of two wheels on a shaft, the velocity at the circumference of the smaller wheel would be reduced, and power gained. By various combinations of wheels thus connected, it is possible to modify the force transmitted to almost any extent.

There is a very ingenious modification of the wheel and axle known as the **Chinese windlass**, or differential wheel and axle, which is represented in fig. 29, and to which we must briefly refer. The cylinder is composed of two portions, one having a larger diameter than the other. The rope is coiled in opposite directions on the two parts of the cylinder, so that while being coiled on the one it is being uncoiled from the other. The distance through which the weight will be moved by one turn of the handle will, therefore, be equal to the difference between the circumference of the larger and that of the smaller portion of the cylinder, which difference can be made as small as required. When the two parts are of nearly the same diameter, there is obviously a great loss of speed; and, as would be expected, a very considerable gain in power. The result is the same as if the handle were connected with an axle of very small diameter, whereas the weakness attending the latter arrangement is avoided in the Chinese windlass.

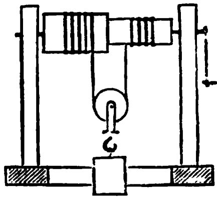
Fig. 29.

EXERCISES.

1. Draw a diagram to represent a wheel and axle in which the power and resistance both act on the same side of the fulcrum or pivot.
2. Has the weight of the wheel and axle itself any effect in either aiding or obstructing the action of the power?
3. If the rope on the windlass is allowed to coil on itself, what difference will it make in the relation between the power and the resistance?
4. A windlass has often two handles attached to the cylinder, one at each end, so that when one handle is at its highest, the other is at its lowest point. Explain the advantage of two handles, and also of their being arranged in the manner described.
5. If the radius of the wheel is 1 yard, and of the axle 6 inches, find the ratio of P to R.
6. The arm of a crank is 5 inches, and the radius of the wheel to which it is attached is 2 feet; what force at the circumference of the wheel will be equivalent to a force of 120 lbs. acting on the crank?
7. A wheel and axle is in equilibrium with a power of 80 lbs. and a weight of 144 lbs.; the radius of the wheel is 16 inches, find the radius of the axle.
8. What arrangement of wheel and axle would be required in order that a force of 112 lbs. might just support a ton?
9. The circumference of the barrel of a windlass is 36 inches, and the weight to be raised is 480 lbs.; what power will be required at the end of a winch (handle) 2½ feet long?
10. A weight of 168 lbs. has to be raised from the bottom of a shaft, by means of a force of 112 lbs.; what relation must the wheel and axle bear to each other? If the shaft is 150 feet deep, through what distance must the power move to raise it?
11. If the power moves 6 feet, while the weight rises 3 inches, what is the ratio between the power and the weight?
12. Two horses are attached to the arms of a capstan, and each exerts a force of 840 lbs.; what load will they raise if the arms are 4 yards long, and the axle has a radius of 18 inches?
13. By using a force of 14 lbs. I can just support a weight of 1½ cwt. on a wheel and axle. The radius of the axle is 6 inches, find the radius of the wheel. Is the power of 14 lbs. applied to the wheel or the axle?
14. Describe an arrangement of wheel and axle, by which the velocity of the body moved would be 10 times as great as that of the power.
15. Find the pressure upon the axis of a wheel and axle when a power of 18 lbs. is in equilibrium, with a resistance of 162 lbs. (*a*) when both forces act in the same direction, (*b*) when the forces act in opposite direction. Illustrate your answer by diagrams.
16. The circumference of the barrel of a windlass is 30 inches, how many times will it turn round in raising a bucket from a depth of 30 feet? If the bucket and its load weigh 200 lbs., through what distance will the power of 25 lbs. move?
17. I pull with a force of 32 lbs. on the rope attached to an axle having a diameter of 12 inches; the wheel connected with the axle has a diameter of 30 inches. What weight hung from the wheel can I support, and through what space will that weight have moved when I have pulled 6 feet of rope off the axle?
18. A windlass has a barrel with a diameter of 8 inches, and a handle 30 inches long; what force will be required to lift a weight of 360 lbs.? The rope has a diameter of 1 inch; what power will be required to raise the weight when the rope begins to coil back on itself?

LESSON 33.

THE PULLEY.

When a body is hung from a cord, the weight of the body stretches the cord, and produces on it a certain **tension** or tightness. The force of tension of the cord is equal to the weight of the body, and acts in the opposite direction to that in which the gravitation of the weight acts. And, if we neglect the weight of the cord itself, the tension of the cord is the same in every part. If the cord is passed over a perfectly smooth bar, as in fig. 30, *A*, instead of being allowed to hang vertically, there is no difference in its tension; for the force applied at the end of the rope to support the weight will be the same as in the former case, only that now it will act in a different direction. This arrangement, therefore, gives no mechanical advantage, but the change in the direction of the force might be a matter of convenience. If the bar were not perfectly smooth—and no bars are—the friction of the rope against it would partly assist in supporting the weight, but would render it necessary to use a much greater force to draw up the weight. The rigidity or stiffness of the rope would cause a loss of more or less of the power, which would be expended in doing the work of bending the rope.

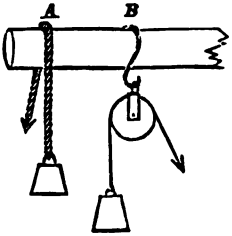

Fig. 30.

If the rope, instead of being passed over the bar, were passed over the circumference of a wheel or pulley hung from the bar, and free to move (fig. 30, *B*), there would be a very much smaller loss of power due to friction. Such an arrangement constitutes what is known as a **fixed pulley**—fixed, that is, in the sense of always occupying the same position. There is no mechanical advantage in the use of a fixed pulley, that is to say, there is no gain either in power or speed, but it is in many cases a very convenient arrangement. It furnishes a very simple means of changing the direction of action of a force, so that the force may be applied in the most convenient manner. By its means a man may stand on the ground and raise a quantity of bricks, or a weight of any substance, to a considerable height, thus saving himself the necessity of raising in addition the weight of his body to the same height. The fact that the tension of a cord remains the same, however much its direction may be changed, provided that it can move without friction, and that we neglect its weight, is most important in connection with the theory of the pulley. Under the term cord we include both ropes and chains, and any other body which is flexible and will bend freely.

If a body is supported by two cords, each will bear part of the weight of the body; and if the cords are of the same length, and similarly supported parallel to each other, the tension of each will be equal to half the weight of the body. And, in general, if a body is suspended by any number of cords equal and parallel to each other, the tension of each cord will be equal to the weight of the body divided by the number of cords, while the sum of the tensions will be equal to the weight of the body. If the cords are not parallel the tensions will be greater than according to this rule, since a portion of the force exerted will be wasted by the cords pulling against each other. If one end of a cord be fastened to a fixed beam, and the cord be passed round a pulley connected with the body (fig. 31), a force equal to half the weight of the body exerted at the free end of the cord will be sufficient to support the body. The weight is then borne equally by the two portions of the cord, and each will have a tension of 1 as compared with the weight of the body 2. There is therefore a mechanical advantage in the use of such a **movable pulley**, for it not only changes the direction of the force, but increases the power. The principle of work applies to this simple form of pulley, as also to the more complicated arrangements of pulleys. To raise the weight in fig. 31 one foot, it is obvious that each portion of the cord must be shortened by that amount, and, therefore, that the free end of the rope must be raised by the power two feet; so that what is gained in power is lost in speed.

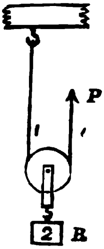

Fig. 31.

There are many ways in which two or more pulleys may be arranged to work together, so as to enable a small power to balance a much greater resistance. In every case there are either several cords, or the same cord is passed round the pulleys several times, so that the weight of the body is distributed among these various supports. And it is this reduplication of the cord which is the characteristic feature of the pulley. The wheel round which the cord passes is technically known as a **sheave**, and the support on which the axis of the wheel rests is known as the **block**. One or more sheaves may be combined in the same block, and the block may be either fixed or movable.

We will now proceed to consider briefly a few examples of combinations of pulleys, and to ascertain the conditions of equilibrium in each case, neglecting the weights of the pulleys and cords themselves, the friction, and the rigidity of the cords. It need scarcely be pointed out that these considerations, which we are now omitting for the sake of clearness, have to be taken into account in the practical application of the sets of pulleys.

Three combinations of pulleys, represented in fig, 32, *A*, *B*, *C*, are usually described in books on mechanics as the first, second, and third systems of pulleys respectively, and we shall adopt this classification for convenience of reference.

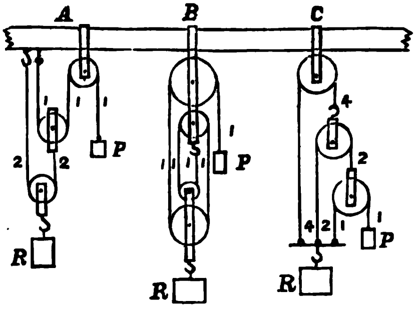

Fig. 32.

In the first system (fig. 32, *A*) there are several cords, each being fastened by one end to the beam. The fixed pulley, as we have explained, simply changes the direction of the force, but the two movable pulleys change also the magnitude of the force. The figures in the diagrams by the side of the cords indicate the relative tension of each cord. Thus the tension of the cord to which the power is applied is marked 1 (that is, $P \times 1$); the movable pulley round which the cord passes pulls on the cord below it with a tension of 2 (that is, $P \times 2$); while the second movable pulley pulls on the weight R with a force of 4 (that is, $P \times 4$). Each movable pulley, therefore, doubles the magnitude of the force. With one movable pulley, $P : R :: 1 : 2$; with two movable pulleys, $P : R :: 1 : 4$, &c. The general rule for this system of pulleys is as follows :—if we call the number of movable pulleys n, then

$$\frac{P}{R} = \frac{1}{2^n} \text{ or } R = P \times 2^n$$

By 2^n is meant the number obtained by multiplying together as many 2's as is represented by n. Thus if n stands for 5, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.

In the second system of pulleys (fig. 32, *B*) the same cord passes round all the pulleys, and therefore has the same tension in every part, viz., a tension equal to the power applied. In the figure there are four pulleys altogether, and four strings attached to the lower block and supporting the weight; and, therefore, the weight supported will be equal to 4 P. As a general rule, if n stands for the number of strings at the lower block, then

$$\frac{P}{R} = \frac{1}{n} \qquad \text{or } R = P \times n$$

In practice, the pulleys in this system are not arranged vertically as in the diagram, but the sheaves in each block are arranged side by side, an arrangement which admits of the weight being raised more nearly up to the beam. And in this latter form this system is perhaps the most useful of all combinations of pulleys, since it allows of a weight being raised through a considerable distance; whereas with the other systems of pulleys, before the weight has been lifted far, further progress is stopped by one of the pulleys having reached the beam or the weight. The form of pulley represented in fig. 13, page 85, belongs to the second system.

The third system (fig. 32, *C*) comprises several cords, each one of which has one end attached to the weight; and the sum of the tensions of the strings so attached (which it will be observed is different for the different strings) is equal to the resistance. The general rule is $\frac{P}{R} = \frac{1}{2^n - 1}$, or $R = P \times (2^n - 1)$ where *n* stands for the number of pulleys (including the fixed one).

There are two other arrangements of pulleys sometimes employed, which are known as Spanish Burtons. These are represented in fig. 33, and it will be an exercise for the student to ascertain the conditions of equilibrium in each case. This can best be done by marking in the tension of the cords, commencing with that to which the power is applied.

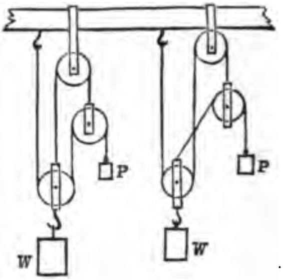

Fig. 33.

The pulley is one of the most convenient of the mechanical powers, and is largely employed in building, in the rigging of sailing ships, and for many other purposes. The loss of power due to friction, and the rigidity of the ropes, is, however, as a rule, very considerable. This loss is less in the case of a large pulley than it is when a small one is employed.

EXERCISES.

1. Find the total pressure on the beam in each of the three systems represented in fig. 32; and in the first system find the pull of each separate thing on the beam.
2. In each system of pulleys state whether the weights of the pulleys will help or hinder the action of the power.
3. What difference is there between the pulley, and the wheel and axle?
4. A weight of 1 ton is supported by 14 equal and parallel cords, find the tension of each cord.
5. A weight of 56 lbs. is raised by means of a fixed pulley; what is the power employed, and what is the total pressure on the axis of the pulley?
6. In a certain arrangement of pulleys, the power of 10 lbs. moving through a distance of 32 feet raises the weight through a distance of 1 foot; what is the weight? What arrangement of pulleys would fulfil these conditions?
7. Sketch an arrangement of pulleys by means of which a power of 2 lbs. will support a weight of 30 lbs.
8. In the first system, if there are 4 movable pulleys, what power will be required to support a weight of 4 tons?
9. In the second system, how many pulleys are there in the lower block when 168 lbs. support 18 cwt.?
10. In the third system, if a power of 1 lb. can support a weight of 31 lbs., how many pulleys are there?
11. If in the first system a power of 28 lbs. can support 2 cwt., how many pulleys are there? Sketch the arrangement.

12. In the second system there are 3 pulleys in each block, and a weight of 42 lbs. to be supported; what power will be required (*a*) if the cord start from the upper block; (*b*) if it start from the lower?
13. How much string is pulled over the fixed pulley in using the first system with 3 movable pulleys in order to raise the weight 1½ feet? If the power is a force of 28 lbs., how much work is done in raising the weight that distance?
14. In the second system, with 2 pulleys in each block, how far would the weight be raised when 29 feet of rope were pulled over the pulley? If the power were a force of 25 lbs., what weight would be raised, and what would be the total amount of pressure on the beam?
15. In the first system, if there were 1 movable pulley weighing 4 lbs., carrying a weight of 16 lbs., what power would be required to maintain equilibrium, and with what force would the beam react on the cord attached to it?
16. Find the relation between P and R, in the Spanish Burtons represented in fig. 33.

LESSON 34.

THE INCLINED PLANE.

An **inclined plane** is the name given to any plane surface, the direction of which makes an angle with a horizontal plane. The amount of inclination of the plane is sometimes expressed by the number of degrees in the angle included between the inclined plane and a horizontal plane. Thus the inclined plane represented in fig. 34 has an inclination of 30°. Sometimes the inclination is expressed by stating the number of units of length along the plane which it would be necessary to traverse in order to rise one unit in height. Thus the plane in fig. 34 has an inclination of 1 in 2, that is to say, when a body is moved 2 feet up the plane it rises 1 foot in vertical height; or the vertical elevation is 1 yard for every 2 yards measured up the plane. The latter method is very commonly employed amongst engineers to express the inclination of the "gradients" (or inclines) on roads, railways, &c. The distance measured along the plane (AB, fig. 34) is called the **length** of the plane; the corresponding horizontal line, AC, is the **base** of the plane; while the vertical line, BC, joining the two, is called the **height** of the plane.

The weight of a body is a force always acting vertically downwards from the centre of gravity of the body. When a body rests on a horizontal plane, this force, therefore, acts perpendicularly to the plane, and the reaction of the plane (which is equal to the pressure on the plane) is of itself sufficient to support the body. When a body hangs in contact with a vertical plane, the weight of the body acts parallel to the plane; there is therefore no pressure on the plane, and no reaction to support the body, which must consequently be supported by an applied force equal to the weight of the body. An inclined plane occupies an intermediate position between these two extremes; a body resting on such a plane does exert pressure on it, and is in part, but not wholly, supported by the reaction of the plane. To support a body on an inclined plane, it is therefore necessary to apply a force in addition to the reaction of the plane, and the force so applied may be considerably less than the weight of the body. The weight of the body resting on the plane in fig. 34 is represented by the line *aw*, which force may be resolved into (*i.e.*, considered as made up of) two other forces, one, *ab*, acting down the plane, and another, *ac*, acting perpendicularly to the plane. The component *ac* represents the pressure on the plane, and will be balanced by the reaction of the plane. The component *ab* urges the body down the plane; and in order to prevent the body from moving, an additional force must be applied to counteract *ab*, —for example, a force equal and opposite to *ab*. It will readily be seen, by taking planes having different inclinations, and resolving the forces after the manner shown in fig. 34, that the greater the inclination of the plane, the greater is the force urging the body down the plane (corresponding, therefore, to *ab*), and the smaller the pressure on the plane. So that the greater the inclination of the plane, the greater the force required to support a body on it. The force supporting the body may be applied in almost any direction, but we shall confine our attention to the two cases where the force acts either parallel to the plane or horizontally, and ascertain the conditions of equilibrium in these two cases.

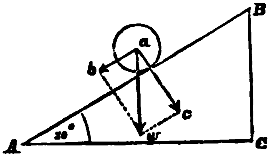

Fig. 34.

A perfectly smooth body, placed on a perfectly smooth and rigid plane, would not remain at rest if the plane were inclined in the least degree from the horizontal direction, but would move down the plane, unless specially supported. We may, however, constantly see stones and other bodies resting on inclined surfaces (such as the side of a hill) without special support. This is due to friction, which counteracts the force tending to move the body down the plane. In the following remarks we shall neglect friction, and assume that the plane is perfectly smooth, and also perfectly rigid, so as not to be bent or indented in the least by the pressure of the body.

The conditions for the equilibrium of a body on an inclined plane may very readily and conveniently be deduced from the principle of work. Let us first take the case where the force acts along the plane, that is, parallel to the length of the plane. This is represented in fig. 35. The power *P* will be less than the weight *W*, since the plane assists in supporting the weight. Suppose the force *P* to move *W* from *A* to *B*, that is, over the length of the plane. In so doing, *W* would be lifted through a vertical distance equal to *BC*, and therefore the work spent on *W*

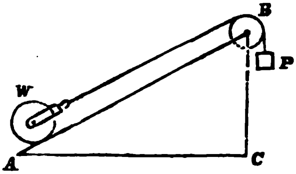

Fig. 35.

would be $W \times BC$. At the same time P would move through a distance equal to AB, and therefore the work done by P would be $P \times BC$. By the principle of work these two amounts of work are equal, or $P \times$ length of plane $=$ $W \times$ height of plane. This may be represented as follows :—

$$\frac{P}{R} = \frac{\text{height of plane.}}{\text{length of plane.}}$$

It can be shown that the pressure on the plane in this arrangement is to the weight, as the base of the plane is to the length of the plane.

When the force is applied horizontally, as is often the case in rolling a barrel up an inclined board into a dray, or in wheeling a barrow up an inclined plank, the conditions are somewhat different. This case is represented in fig. 36, where the power P, represented by the line, acts horizontally on the weight W. Since the work done by a force is found by multiplying the force into the distance through which its point of application is moved, in the direction in which it acts, the work done by P in moving the body from A to B, is equal to $P \times AC$ (that is, $P \times$ base of the plane). The work done on W in raising it from A to B through the vertical height BC, is, as in the former case, $W \times BC$. Therefore, since these amounts of work are equal, it follows that $P \times$ base of plane $=$ $W \times$ height of plane, or

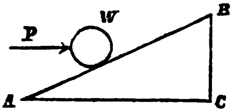

Fig. 36.

$$\frac{P}{R} = \frac{\text{height of plane}}{\text{base of plane.}}$$

The pressure on the plane in this arrangement is to the weight, as the length of the plane is to the base of the plane.

It will be observed that, as the length of the plane is always greater than the base, the greater mechanical advantage is obtained by applying the power along the plane ; as is done when a horse draws a cart up a hill.

The reason for including the inclined plane among the mechanical powers is now apparent ; by its means a small power is able to overcome a greater resistance. In practice no planes are perfectly rigid and smooth, and therefore the rules we have stated above are only approximately true for such planes. The friction on inclined planes is not, however, without its advantages ; for were our inclined surfaces devoid of friction, it would be impossible for us to walk up a hill or slope, or for a horse or train to ascend even the smallest gradient.

The simple character of the inclined plane renders it of very great importance in its practical applications. The stairs and ladders by means of which we raise ourselves to moderate heights above the ground are modified inclined planes ; the steps of the one and the rounds of the other being merely devices to prevent us from slipping, as we should otherwise do on planes so steeply inclined. The small sloping ascents often seen on each side of a bridge are inclined planes, by means of which a person or an animal can easily raise themselves to the height of the bridge. The sloping planks or ladders up which heavy stones, boxes, barrels, &c., are pushed into carts, are likewise applications of the principle of the inclined plane.

EXERCISES.

1. Find the power required to support 1 cwt. on a plane whose height is 6 ft. when the length is 7 yards.
2. If the power is 35 lbs., height of the plane 15 inches, length 37 inches, find the weight.
3. A force of 28 lbs. is sufficient to raise 190 lbs. to the vertical height of 4 feet on an inclined plane without friction; through what distance along the plane does the weight move?
4. A body weighing 3 tons rests on an incline which rises 3 feet in 56, and is without friction; with what force is the body urged down the plane?
5. A power of 75 lbs. is exerted along a plane 5 yards long and 3 feet high; what weight will it raise?
6. A power of 20 lbs. will raise 600 lbs. to a height of 4 ft. on an inclined plane to which it acts parallel, what is the inclination of the plane?
7. A railway train weighing 50 tons is supported on an incline rising 1 in 50, by a rope attached to a stationary engine; find the tension of the rope.
8. An incline is 1 foot in vertical height for every 15 feet in length; a weight of 100 lbs. resting on the plane is just beginning to slide, but is prevented by friction; find the amount of friction.
9. A weight of 25 tons is supported on an inclined plane rising 1 in 40 by a rope and 6 horses; with what force must each horse pull?
10. A board rests on a cart 3 feet high, and its lower edge is 14 feet from the cart, what weight will a power of 54 lbs. lift if applied horizontally?
11. A force of 180 lbs. is applied parallel to the horizon to raise a weight of 990 lbs. on to a platform 3 ft. high; how far must the lower edge of the plane be from the foot of the platform?
12. A force of 27 lbs. is applied horizontally to raise a weight to the top of a wall 4 feet high, from a point 30 ft. distant; what is the weight?
13. A stone weighing 360 lbs. is raised into a cart 3 ft. from the ground by means of an inclined plank 5 ft. long. Draw a diagram to scale, to represent the inclined plane, and find by measurement the length of the base of the plane, and find what power would be required to raise the stone (*a*) acting horizontally; (*b*) acting along the plane.
14. A brick weighing 6 lbs. lies on a plank 8 feet long, one end of which rests on the ground and the other at a height of 50 inches; how great must the friction be to keep the brick from slipping?
15. A stone is laid on a roof having a slope of 2 in 11, and the friction is equal to one-eighth of the weight. Will the stone slide down?
16. Two inclined planes stand back to back; one having an inclination of 2 in 9, and the other 2 in 5. A weight of 54 lbs. rests on the former, and is supported by a string passing over a fixed pulley at the summit of the planes, and connected with a weight on the second plane. Find the weight.

LESSON 35.

THE WEDGE.

The **wedge** is an instrument whose action depends on much the same principle as that of the inclined plane. But while the inclined plane itself is stationary, and the body is moved along it, the wedge itself is moved; and we may regard the wedge as a movable inclined plane. Its usual form is shown in fig. 37, *W*, where it is seen to be a block having a triangular section, the angle at the apex of the triangle varying very considerably, according to the amount of resistance to be overcome, and other circumstances. The wedge may be regarded as a double inclined plane, composed, that is, of two inclined planes base to base, and having the power applied to the backs (or heights) of the planes. It would not be difficult, according to this view, to state the theoretical relation between the power and the resistance in the case of the wedge. But the friction encountered in the use of the wedge is so very great, that the theoretical conditions of equilibrium are not even approximately true, and therefore there is no practical advantage in determining these conditions. There is also another reason why the theoretical rule for the action of the wedge cannot readily be applied in practice, viz., that in consequence of the great resistance which a wedge has to encounter, it is moved by percussion (*i.e.* by striking it with a hammer, &c.), and not by a steady pressure, as is the case with the other mechanical powers. And it is difficult to compare the force of a blow, with the steady continuous action of a force, so as to be able to express them both in the same units. The difference between a blow and a steady force is well illustrated, by the difficulty of breaking an egg by crushing it steadily at the ends, while it is an easy matter to break it by means of a slight blow from a spoon. A blow, in fact, is something like a condensed force, *i.e.*, a considerable amount of force compressed into a short space of time.

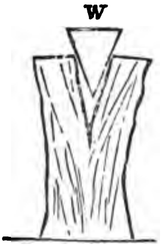

Fig. 37.

The friction which so powerfully resists the motion of a wedge is not without its compensating advantages; and, indeed, a wedge could not be used at all in the ordinary manner were it not for friction. For when, by means of a blow, the wedge has been moved a short distance in spite of the resistance, the friction is so great as to prevent the wedge from slipping back again, so that each advance is so much gain.

When we consider the various applications of the wedge, we see that it is especially useful where large resistances have to be overcome through short distances only. Thus, if a very large block of stone resting on the ground is to be raised, for the purpose of passing a chain or rollers under it, it can be conveniently done by driving several wedges underneath it at different points. In this way a mass of several tons in weight may be raised an inch or more from the ground by the efforts of one man. In the same manner, when a heavy cupboard or case has become bent, and thrown somewhat out of shape by standing on an uneven floor, it can be most conveniently put right by lifting it at the points which have been lowered by means of suitable wedges. To lift a heavy body to the height of a few inches only by means of wedges would be a very inconvenient arrangement; the value of the wedge as a mechanical power consists, as we have said, in overcoming great resistances through short distances. For example, it requires a considerable force to tear the fibres of wood away from each other, but if they are separated even to a very small distance the force of cohesion is overcome. Hence the wedge is especially adapted to the splitting of blocks of wood and similar substances. An axe is nothing more than a wedge having a very sharp edge, and the same is true of most of our cutting instruments,—knives, chisels, planes, &c. The angle given to the end of the wedge varies somewhat with the nature of the substances on which it has to be used,—being less, for example, when used on wood than when used on metal; but, other things being equal, its mechanical advantage increases as the angle diminishes.

EXERCISES.

1. Give some more examples of the application of the wedge.
2. A wedge is frequently driven into the head of the handle of a hammer, pick, &c. ; what is the principle on which it acts?
3. Why is the mechanical advantage of a wedge increased by diminishing its angle?

LESSON 36.

THE SCREW.

An inclined plane does not necessarily extend in a straight line, but may wind round and round spirally like a spiral staircase, or like a path winding round the side of a hill. It is much easier to ascend a steep hill by means of an inclined pathway winding round its sides, than to pass in a direct line from the base to the summit. The inclined planes in the two cases have both the same height, but the length of one is very much greater than that of the other, and therefore the power required to raise a given weight is much less in the former than in the latter case. By winding it spirally an inclined plane of considerable length may be made to take up but little room. The winding staircase, by means of which a high tower is ascended, would extend to a considerable distance from the tower, if extended in a straight line. When the spiral plane is movable on an axis passing down its centre, it constitutes a **screw**, which is simply a movable spiral inclined plane. The spiral line traced out by the edge of the inclined plane (fig. 38, *a*, *b*, *c*, *d*), and which constitutes the ridge projecting from the axis of an ordinary screw, is called the **thread** of the screw, and may be either square or V shaped in section. The distance from one point in the thread to the point vertically over or under it in the next succeeding turn (as from *b* to *a*, or *b* to *c*), is called the **pitch** of the screw. A screw with a pitch of a quarter of an inch has its threads that distance apart, and has 4 threads to the inch.

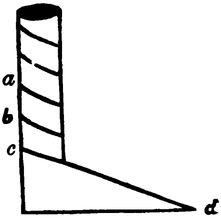

Fig. 38.

A screw works in a corresponding hollow screw known as a **nut**, the thread of the one fitting into the corresponding hollows of the other, fig. 39. The steel screws used for woodwork are made with sharp edges, so that they may cut their own nuts in the comparatively soft material. If the nut in which a screw works is fixed while the screw turns, then the screw moves backward or forward, according to the direction in which it is turned. And in one complete revolution, the screw moves through a distance in the direction of its length equal to its pitch; that is, a screw having six threads to the inch, would move one-sixth of an inch for one turn, or 1 inch in 6 turns. When the screw is arranged so as to turn round, but not to move in the direction of its length, the nut is left free, and moves a distance equal to the pitch of the screw for each revolution. In one form of drilling tool the drill is connected with a screw, which is rapidly rotated alternately in opposite directions by moving a loose nut up and down it. A lever handle is frequently attached to the screw, as represented in the figure, so as to increase the mechanical advantage of the instrument. The power applied at the end of the handle has to move through a much greater distance than it would do if applied directly to the circumference of the screw itself, and there is a corresponding gain in power.

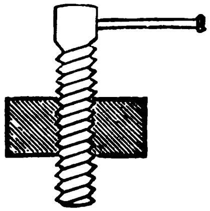
Fig. 39.

The theoretical relation between the power and resistance in a screw, assumed to work without friction, is readily obtained from the principle of work. While the power describes the circumference of a larger or smaller circle, according to its distance from the axis, the screw moves, and overcomes the resistance, through a distance equal to its pitch; and the relation between the two forces will be inversely as the distances through which they move, or

$$\frac{P}{R} = \frac{\text{pitch of screw.}}{\text{circumference of circle described by } P.}$$

The distance from the centre of the screw cylinder, or axis of the screw, to the point where the power is applied, forms the radius of the circle described by the power. From this the length of the circumference may be approximately obtained by means of the ordinary rule, viz., to multiply the radius by $6\frac{2}{7}$, or to multiply the diameter by $3\frac{1}{7}$.

In practice it is found that a very much greater power is required to overcome a given resistance, than would be required according to the above rule. This is due to the great amount of friction which is experienced where the screw thread comes into contact with the nut; and this is especially the case when there is a great resistance to be overcome, and therefore a great pressure on the nut. In some cases the friction is so great as to reduce the efficiency of the screw to one-fourth of its theoretical value. The friction is useful, however, as in the case of the wedge, for it prevents the screw from slipping back after it has been moved. The value of ordinary screws and bolts in drawing pieces of wood and iron firmly together, and keeping them in that position, depends upon this friction. The screw is a most valuable instrument where a great resistance has to be overcome through a short distance, as, for example, where a powerful pressure has to be brought to bear on a body. But where a smaller resistance has to be overcome through a great distance, as where a body is to be raised to a great height, the screw is not so serviceable as some of the other mechanical powers.

The ordinary screw-press and the common vice are familiar examples of the application of the principle of the screw. In both these instruments a great pressure is required to act over a short distance only, and therefore a screw combined with a lever handle is specially adapted for the purpose. The mechanical advantage of a screw for these purposes may be increased, either by lengthening the lever handle, or by using a screw of a smaller pitch. There is, however, a limit to the diminution of pitch in screws which have to overcome great resistances, since if there are many threads to the inch, each thread must be attached to the screw cylinder by a very narrow base, and therefore there is a tendency for the threads to be broken off by great pressure.

Nearly all large ships are now fitted with a screw-propeller, that is, an axis fitted with two vanes or arms, forming parts of a large screw surface. The revolution of this screw acts on the water as on a nut, and the reaction due to the resistance of the water moves the vessel.

The screw is often employed for the measurement of minute distances, and in this form it is known as a "micrometer screw." Suppose, for example, that a measuring gauge has one of its measuring surfaces movable, and connected with a screw having 10 threads to the inch. If a wheel is connected with the screw, and the circumference of the wheel is divided into 100 divisions, a movement of the wheel through one division (as indicated by a small fixed pointer standing over the graduations on the wheel), will correspond to a movement of the end of the screw through the $\frac{1}{1000}$th of an inch, viz., $\frac{1}{100} \times \frac{1}{10}$. By the use of such an instrument, a difference of the $\frac{1}{1000}$th of an inch in the length of two bars could be ascertained and measured. Micrometer screws are connected with many astronomical instruments, and with others in which means for very accurate measurement are required.

Another modification of the screw is known as the worm, or "endless screw." This is represented in fig. 40, and consists of an axis on which there are several turns of a screw thread, which, as the axis revolves, engage with some of the teeth on the circumference of a wheel, and keep the latter in motion. This wheel, therefore, occupies the place of the nut in the ordinary arrangement of the screw.

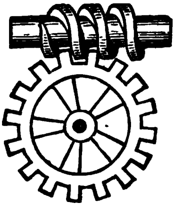

Fig. 40.

EXERCISES.

1. Why are the ordinary nuts for bolts made with several flat faces on their outer surface?
2. Explain the advantage of a screw-driver.
3. Mention some common boring instruments which work on the same principle as the screw.
4. Find the relation between the power and resistance in a screw which has 12 threads to the inch, and is moved by an arm 8 inches long.
5. A screw has a pitch of 1 inch, and is moved by a lever 1 yard long; what pressure will be produced when a force of 56 lbs. is applied to the end of the lever?
6. The pitch of a screw is 1 inch, and by means of it a weight of 1000 lbs. is to be raised by a force of 3 lbs.; what should be the length of the lever?
7. A screw is employed to produce a pressure of 1 ton when acted on by a power of 5 lbs. at the end of a lever 4 feet long, find the pitch of the screw.
8. A force of 15 lbs. applied to a lever 4 feet long moves a screw having 6 threads to the inch; what amount of resistance can be overcome, and through what space must the power move in order that the screw may be advanced one inch?
9. What power will raise 1,432 lbs. when the screw has 11 threads to the inch, and the power describes a circle 2 feet in circumference?
10. A screw advances 2 inches in 21 turns, and the resistance to be overcome is 1000 lbs. What power must be applied at the end of a lever 18 inches long?
11. A bolt with 10 threads to the inch is employed to draw two iron plates together, the nut being turned by a wrench 1 foot long. What power will be required to press the surfaces together with a force of 1 ton if there is no friction?
12. The screw of a small vice has 12 threads to the inch, and is worked by a handle 6 inches long. The power of the screw is applied at a distance of 5 inches from the fulcrum to a lever of the third order, while the resistance is at a distance of 6 inches. What will be the pressure on the body in the vice when a power of 12 lbs. is applied to the lever?

LESSON 37.

COMPOUND MACHINES.

The six mechanical powers which we have considered in previous lessons are constructed upon three distinct principles. These are (1) the principle of the lever; (2) the principle of the pulley; and (3) the principle of the inclined plane. The lever and the wheel and axle are arranged on the first of these principles; the pulley is the only one acting on the second principle; while the inclined plane, wedge, and screw, each embodies the third principle. As we have before stated, ordinary machines are composed of parts, which are seen to be mechanical powers, either in their simple form or slightly modified, together with the necessary rods, chains, and other arrangements employed in connecting the working parts together. In the present lesson we will briefly consider a few simple examples of combinations of the mechanical powers.

A number of levers may be arranged together, so that when a power is applied to one of them it may bring the whole into action. Thus in fig. 41 three combined levers are shown; the small arm of the first, to which P is applied, acting on the longer arm of the second; the shorter arm of which, in its turn, acts on the longer arm of the third lever, the resistance being applied to the shorter arm of this same lever. The arms of the levers are respectively 10 to 1, 8 to 1, and 3 to 1; and the mechanical advantage is $\frac{10 \times 8 \times 3}{1 \times 1 \times 1} = 240$.

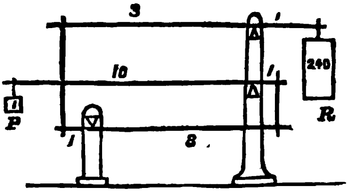

Fig. 41.

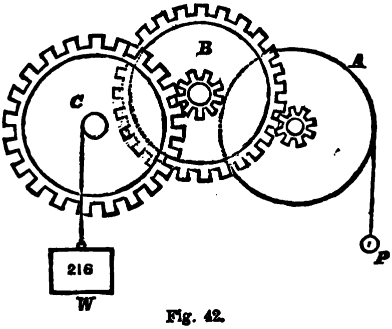

Fig. 42.

In a similar manner a number of wheels may be made to communicate motion to each other, either by means of belts or of teeth; and if the wheels are arranged on the principle of the wheel and axle, a very considerable mechanical advantage may be obtained. Thus, in fig. 42, three pairs of wheels are connected, the smaller wheel of each pair driving the larger wheel of the succeeding pair. Suppose that the radius of each of the large wheels is 18 inches, and of each of the small wheels 3 inches. Then calling the power applied to *A*, one, the power applied to the large wheel *B* will be increased to 6, and to the large wheel *C* to 36 (viz., 6 × 6), while the power applied to raise *W* will be 216 (viz., 6 × 6 × 6). Such a train of wheels might also be employed to increase the velocity of any movement, in which case it would be necessary to apply the power to the axle from which *W* is suspended in the figure. If the works of a clock are examined, it will be seen that they consist for the most part of wheels working on each other, somewhat in the manner of those represented in fig. 42. The power derived from the weight, or the spring, is transmitted to the axle connected with a large toothed wheel, which engages with the axle of another pair of wheels; and in this manner the very slow movement of the weight or spring is converted into the very much faster movement of the minute hand.

The mainspring which drives the works of a watch exerts a force, which varies in magnitude with the extent to which the spring is coiled. When the spring is fully wound up, it acts with a much greater force than when it is partly uncoiled, and in fact the force it exerts as it uncoils gradually and constantly diminishes. The inequality which would otherwise ensue in the working of the watch is avoided by an ingenious arrangement known as the fusee, fig. 43. This is a conical barrel *F*, with a groove cut spirally round its circumference, and in this groove the chain connecting the fusee with the barrel *S*, containing the spring, is coiled when the watch is wound up. When the spring begins to uncoil, and exerts its strongest force, it acts on an axle of small diameter; while as it uncoils, and its power becomes diminished, the portion of the wheel on which it acts becomes constantly larger. The varying force, therefore, acts on a lever with varying length of arm, the leverage increasing in the same proportion as the power diminishes. If the spring and fusee are properly adjusted to each other, the effect of the force will be equal at all times.

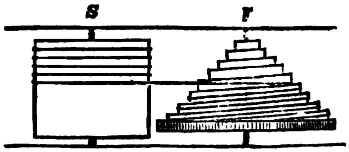

Fig. 43.

The **crane** employed for lifting heavy weights, such as the cargo out of vessels, or the beams and stones required in building, is worthy of a little study. The principal parts of this machine are shown in fig. 44. There is a strong wooden beam, or sometimes a framework of iron, at the top of which is a fixed pulley. In many cases the chain passing over this pulley from the barrel *C* of the windlass has the weight suspended at

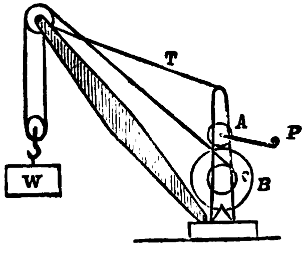

Fig. 44.

its extremity; but in other cases the weight is hung from a movable pulley, and the chain passing round this pulley is fastened to the beam, or "jib" as it is technically called, as is represented in the diagram. The windlass round which the chain coils is either worked directly by a handle, or, as is represented in the figure, the large wheel, *B*, of the arrangement is driven by a small wheel or pinion *A*, connected with the handle to which the power is applied. Here we have, therefore, a combination of the lever, the wheel and axle, and the pulley, and all arranged so as to gain power. Let us see what mechanical advantage is obtained by these arrangements, assuming that there is no friction or other resistance. Suppose that the weight *W* is one ton, the movable pulley will reduce the tension of the chain passing to the windlass to half this amount, viz., 1120 lbs. If the radius of the barrel or axle on which the chain coils is equal to half that of the wheel with which it is connected, this again will reduce the force required by one half, so that a force of 560 lbs. at the circumference of the wheel *B* will suffice to support the ton weight. Lastly, suppose the radius of the small pinion, which drives this wheel, to be equal to one-seventh part of the length of the handle or winch; this will reduce the force required to 80 lbs. Then, if, as is usual, there are two handles, one on each side, and one man to work each handle, a force of 40 lbs. exerted by each man would be sufficient to support the weight of one ton hanging from the chain. When the handles were turned the weight would of course ascend very slowly, the power having to traverse a distance 28 times as great as that traversed by the weight. The rod *R* represented in the figure is known as the "tie rod," which we shall have occasion to refer to in Lesson 39.

The **governor** of a stationary steam engine, invented by Watt, is a simple mechanical arrangement of considerable interest and importance. It consists of two heavy balls which are placed each at the end of a lever, the two levers being connected at their upper ends with a shaft, (fig. 45, *S*) kept in rotation by the engine. These levers are also connected by means of another pair of levers with a loose collar *C*, capable of moving freely up and down the shaft. This collar is again connected, by means of the bent lever *B*, with the rod *V* working the valve which admits steam to the cylinder. An increase in the velocity of the moving parts of the engine, caused, for example, by the removal of some resistance or obstruction, increases the speed with which the balls of the governor rotate. The increased centrifugal tendency makes them move further apart, and thus lift up the collar *C*; which movement has the effect of partly closing the valve, and therefore admitting less steam to the cylinder. When the speed of the engine and of the balls is suddenly diminished, the balls fall lower down

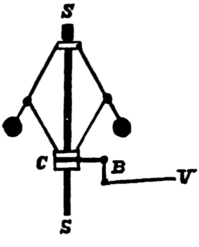

Fig. 45.

and nearer together, thus pushing down the collar, and opening the valve to a greater extent. By this simple arrangement the engine is able to a great extent to regulate itself and maintain a constant speed.

EXERCISE.

Point out the mechanical powers and arrangements comprised in the construction of a coffee-mill, sewing-machine, wringing machine, &c., as far as you can.

LESSON 38.

THE HYDROSTATIC PRESS.

A liquid, when left to itself, takes a horizontal surface, in consequence of the action of gravity upon the particles of the liquid. These particles, or molecules, are so far free from each other's attraction, as to be moved among themselves with the slightest force. Each molecule is acted on by gravity, and has a tendency to move, and it also presses upon any molecules which may be below it, and tends to move them. It is therefore necessary that a liquid should be supported laterally; for if it were not so supported, the molecules in the lower layers of a mass of liquid would move sideways, under the influence of the pressure of the upper layers. If any portion of a liquid were raised above the general level of the remainder, it is obvious that there would not be this lateral support, and the liquid would consequently spread itself out and settle down, until the horizontality of the surface were restored, in which condition each molecule would be laterally supported, either by other molecules or by the sides of the vessel.

The pressure of a liquid acts perpendicularly at every point of any surface with which it is in contact, whatever the form or inclination of the surface may be. If the pressure is due simply to the weight of the liquid, the pressure at any point will vary according to the depth of the point below the surface of the liquid, and also according to the specific gravity of the liquid. Thus, at a depth of about 30 inches below the surface in a quantity of mercury, the pressure of the liquid will be equal to about 15 lbs. on every square inch, while there would be the same pressure at the depth of about 33 feet in water. This pressure is equal to the pressure of the atmosphere at the sea level, and it is usual to speak of a pressure of 15 lbs. to the square inch as a pressure of 1 atmosphere. At a depth of 66 feet in water there would be a pressure of 2 atmospheres, or 30 lbs. to the square inch. In addition to the pressure due to the weight of the liquid, there would also of course be the pressure of the air transmitted by the liquid.

A liquid transmits a pressure applied to it, with equal facility in all directions, and without loss. If a pressure of 1 lb. is applied to a square inch of any portion of the surface of a liquid, it is transmitted equally by the liquid in all directions; so that on every square inch of every surface with which the liquid is in contact there is an additional pressure of 1 lb. A surface of 10 square inches would therefore have a total additional pressure of 10 lbs. put upon it, in consequence of the pressure of 1 lb. applied to the square inch of the surface of the liquid. This is the principle of the hydrostatic press, in which a pressure is applied to a small area of the surface of a liquid by means of a piston, and transmitted by the liquid to a piston having a very much larger area. The magnitude of the force exerted by the liquid on the larger piston is as much greater than that of the force applied by the smaller piston, as the area of the larger piston is greater than that of the smaller.

The **hydrostatic press** (known also as the **hydraulic press**, or **Bramah's press**) consists essentially of a very strong cylinder of cast iron, in which moves a solid piston or "ram" (fig. 46, *R*) surrounded by a water-tight collar at *C*, and carrying a flat table at its top on which the objects to be submitted to pressure are placed. This cylinder contains water, and communicates by means of a small strong pipe with a small force pump, which contains a small piston or "plunger" *P*, worked by a lever handle. When this force pump is worked, water is pumped into the cylinder, and the ram slowly rises, and presses the objects placed on it with enormous force against the fixed beam above. If the area of the smaller piston is 1 square inch, and of the larger piston 1 square foot, the force applied to the smaller piston will be multiplied 144 times. And if the lever gives a mechanical advantage of 6 to 1, a force of 10 lbs. acting on the lever will be converted into a force of 8,640 lbs. (viz., 144 × 6 × 10) on the larger piston. A force of 10 lbs. is therefore enabled by the intervention of this machine to raise a weight of nearly 4 tons. This is on the assumption that the machine works without friction, which of course is not the case. This enormous pressure is exerted not only on the ram, but also on the sides of the cylinder: that is to say, every square foot of the interior surface of the cylinder is pressed by the water with a force of nearly 4 tons. Under this enormous pressure there is a great tendency for the water to leak out by the side of the ram, and for a long time no means could be found of overcoming this difficulty. Bramah suggested a kind of half tube of leather, forming a collar round the ram, with its open part directed downwards. The water

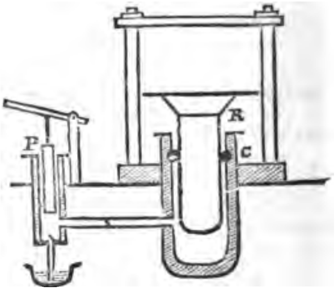

Fig. 46.

makes its way into this opening, and presses the leather with immense force against the ram on the one side and the cylinder on the other, thus forming a perfectly water-tight collar.

The principle of work is readily seen to apply to the action of the hydrostatic press. If the area of the larger piston is 144 times as great as the area of the smaller piston, the quantity of water forced out by the latter in moving down one foot, would find room in the larger cylinder if its piston rose only the $\frac{1}{144}$th part of a foot. In the hydrostatic press, as in machines generally, a gain of power is accompanied by a loss of speed in the same proportion.

The hydrostatic press is a most useful instrument for applying great pressure. Bales of cotton, wool, hay, &c., can be compressed into very small space by means of such presses, so as to occupy but comparatively little room in the vessels by which they are to be carried. Lead pipes are usually made by forcing out the solid lead while soft through a suitable aperture in a steel block, under powerful hydraulic pressure. Large hydrostatic presses were employed by Robert Stephenson in raising the tubes of the Britannia Bridge. One of the largest of these had a cylinder of cast-iron 10 inches thick, and weighing 15 tons, while the ram weighed more than 3½ tons, and was capable of raising a load of 2000 tons.

EXERCISES.

1. A vessel has a base measuring 12 inches by 10 inches, and contains water to a depth of 4½ feet; find the total pressure on the base.
2. A glass vessel contains mercury, find the pressure per square inch on the base of the vessel if the depth of the mercury is 18 inches.
3. A hydrostatic press having a large piston with an area of 16 sq. inches supports a weight of 72 tons; find the pressure on every square inch of the cylinder.
4. In a press the plunger is ⅞-inch diameter, and the ram 9¼ in. diameter; the plunger is worked by a lever having arms respectively 78 inches and 3¼ inches long; show that *W* is about 2,680 times as great as *P*. [The areas of circles are in the proportion of the squares of the diameters.]
5. A lever having arms of 40 and 5 inches respectively works a plunger having an area of ¾ of a square inch; the ram has an area of 90 square inches; what pressure on the ram will be produced by a force of 28 lbs. acting on the lever? Through what space must the plunger move in order that the ram may rise 1 foot?

LESSON 39.

THE PARALLELOGRAM OF FORCES AND VELOCITIES.

In a previous lesson (Lesson 8) we have described the parallelogram of velocities as a rule, by means of which the direction and velocity of the resultant motion of a body, to which two separate forces are imparting velocities in different directions at the same time, may be found. The parallelogram of forces is, similarly, a rule by which to find the direction and magnitude of the resultant of two inclined forces. In the present lesson we will consider a few simple applications of these rules.

Let us, in the first place, apply the parallelogram of velocities to ascertain approximately the path which will be taken by a projectile, shot out in a horizontal direction, and acted on at the same time by the force of gravity. Let the line *AB* (fig. 47) represent the motion of the body for one second, under the action of the force of projection alone; and the line *AD*, at right angles to the former, represent the motion of the body for the first second under the action of gravity alone. The actual path of the body will be shown by the line *AC*. For the next second the motion is similarly found by means of the parallelogram *COEH*, the line *CO* being equal to *AB*, if we neglect the resistance which the air offers to the flight of the projectile, while *CH* is 3 times as long as *AD* for reasons explained in our lessons on gravitation. In the same way, the course during the third second is represented by *EF*, so that the path of the projectile is *ACEF*. As a matter of fact, the downward velocity of a body does not change abruptly at the end of each second as represented in the diagram, but is constantly increasing, and the path described by a projectile, under the circumstances mentioned, is the curve known as a parabola.

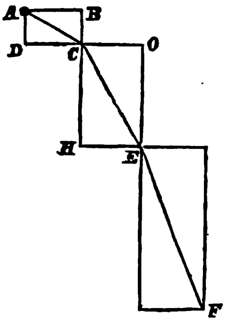

Fig. 47.

In the second place, when a horse moves a barge along a canal by means of a rope, a portion only of the force which the horse exerts is spent in moving the boat forward. This can be shown by means of the parallelogram of forces, as in fig. 48; where the force exerted by the horse is represented by the line *BH*. This is resolved into the two forces *BA*, *BC*, at right-angles to each other; and it is plain that only the component *BA* is effective in moving the boat forward. The other component, *BC*, tends to move the boat sideways, and its effect has to be counteracted by means of the rudder. The portion of the force effective in moving the boat on its course, bears to the total force exerted by the horse the same proportion as *AB* to *BH*.

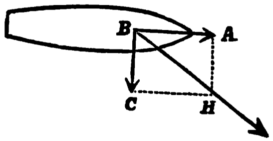

Fig. 48.

The action of the wind on the sails of a ship, when the direction of the wind is not exactly the same as that which it is desired the vessel shall take, may conveniently be studied by the help of the parallelogram of forces. Let *VV′* (fig. 49) represent the vessel, and *SS′* the sail, placed obliquely to the direction *V′V* in which the vessel

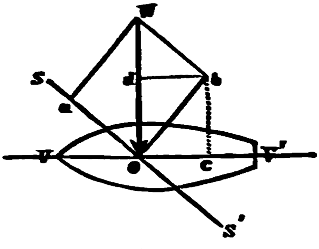

Fig. 49.

is to proceed, and also inclined to the direction of the wind, *Wo*. The wind is supposed to blow in a direction perpendicular to that of the ship's course. The force of the wind represented by *Wo* may be resolved into two parts, viz., *ao* acting parallel to the sail, and *bo* acting perpendicularly to the sail. The former component, it is obvious, can have no part in the motion of the ship, since it produces no pressure on the sail. We may therefore neglect the part of the force represented by *ao*, and confine our attention to the component *bo*. Now it is plain that since this force does not act in the direction of the vessel's course, it is not all effective in moving the vessel in that direction; and therefore we must resolve *bo* (as shown in the diagram by the dotted lines) into two forces, one of which, *co*, acts in the direction of the vessel's course, and the other, *do*, acts in a direction at right-angles to the former. The tendency of the component *do* is to move the vessel sideways, and this is counteracted partly by the great length of the vessel in that direction (and therefore the great resistance opposed by the water), and partly by means of the rudder. So that, in the case we have considered, it appears that part of the force of the wind is directed parallel to the surface of the sail, and is therefore ineffective; part is spent in moving the vessel sideways, and is therefore useless; while only a portion, which bears to the whole the same ratio as *co* to *Wo*, is really effective in propelling the vessel.

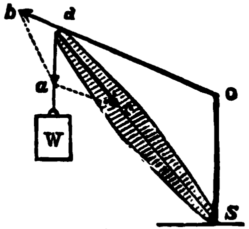

Fig. 50.

The parallelogram of forces can be applied to explain the manner in which the force is distributed between the parts of the crane, and the direction in which each of the parts is strained. In fig. 50 *ds* represents the jib, and *do* the tie-rod attached to the upright support, *os*. A weight, *W*, is supported from the upper end of the jib, by means of the reaction of the jib and tie-rod. Taking the line *da* to represent the weight, that force may be resolved into two others, one, *db*, in the direction of the rod, the other, *dc*, in the direction of the jib. It will be observed that the force acting on the jib is in the nature of a thrust, tending to crush the beam; and as beams of timber are well adapted to resist crushing forces, they are frequently employed to form this portion of a crane. On the other hand, the force *db* acting on the tie-rod is a pull, and this is best counteracted by means of a rod of wrought iron, which substance is especially valuable on account of its tenacity.

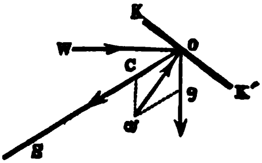

Fig. 51.

A kite when at rest in the air is in equilibrium under the action of three forces, viz., the weight of the kite, the tension of the cord, and the pressure of the wind. These are represented in fig. 51; *Wo* representing the force of the wind, *og* the weight of the kite *KK′*, and *oc* the tension of the cord. The force of the wind *Wo* must be resolved into two parts, one along the kite, and the other perpendicular to it; the latter, represented by *wo*, being the only part concerned in supporting the kite. The force of the weight acts vertically downwards, and it will be seen by the diagram that *wo* corresponds with the diagonal of the parallelogram *ocwg*, and acts in the opposite direction to the resultant of *oc* and *og*.

A further illustration of the application of the parallelogram of forces will be found in the lesson on the inclined plane.

EXERCISES.

1. Why is it a disadvantage to have a very short rope connecting a horse with the barge it is drawing?
2. Draw a diagram to represent a body on an inclined plane supported by a horizontal force, and show by means of the parallelogram of forces the amount of pressure on the plane, and that component of the force which prevents the body rolling down the plane.
3. Resolve the force acting upon a displaced pendulum into two, viz., one tending to move the pendulum to its lowest point, and one producing a tension of the string.
4. Show by means of the parallelogram of forces that when a man draws a heavy stone along the ground by means of a rope, part of the force he applies is useful in raising the body over small obstacles.

EXAMINATION QUESTIONS.

(On the subjects of Lessons 28 to 39.)

1. What is force? Give the names of several forces employed to work machines, with examples.
2. What is "work?" In what units is work measured? How many units of work are done in raising one ton through three inches?
3. Describe some simple experiments to illustrate the transmission of force.
4. A string has attached to it a weight of 4 lbs. at the bottom, 1 lb. at its middle point, and 2 lbs. half-way between the other two weights. Find the tension of the several parts of the string.
5. What is a machine? What advantages are gained by the use of machines? What cannot a machine do?
6. Name the simple mechanical powers, and give an example of the use of each.
7. "What is gained in power is lost in speed." Explain this statement, and illustrate your answer by reference to some particular machine.
8. A force of 10 lbs. applied to a simple machine moves through 1 foot, and raises a weight through ⅛ inch; what is the weight?
9. What resistances to the motion of machines cause a waste of the energy of the power? What becomes of such energy?
10. What is a lever? What is the fulcrum? What are the conditions of equilibrium in the lever?
11. Describe the three kinds of levers, and illustrate your answer by diagrams.
12. Give one familiar example of each of the three orders of levers, pointing out in each case where the fulcrum, power, and resistance respectively are situated.
13. Classify the following levers:—Scissors, bellows, nutcrackers, fire-tongs, a wheelbarrow, a pair of shears, an oar, a man's arm.
14. If a straight lever of the first class, 10 feet long, has a weight, W, at one end, 2 feet from the fulcrum, what power will be required at the other end to balance it? What will be the pressure on the fulcrum?
15. If a lever is 1 foot 4 inches long, the power 8 lbs., and the weight 24 lbs., where must the fulcrum be?
16. How would you arrange a lever of the second order so that a power of 15 lbs. could support a weight of 75 lbs.?
17. If a nut can resist a force of 4 lbs. only, and I press with a force of 3 lbs. on the ends of a pair of nut-crackers 6 inches long, at what distance from my hand must the nut be placed so as just to crack?
18. Draw a diagram to represent a lever of the third order, and give two examples of the use of a lever of this kind. What advantage is gained by a lever of this order?
19. The power acts on a lever at a distance of 2 feet from the fulcrum, and the resistance is on the same side of the fulcrum, and at a distance of 9 feet from it, what is the relation between P and R? When the power moves 1 foot, through what distance will the resistance be moved?
20. To what order of levers does the steelyard belong? For what purpose is it used? If the movable weight on a steelyard be 5 lbs., and it be placed 18 inches from the fulcrum to balance a bag of flour placed ¾ inch from the fulcrum, find the weight of the flour.
21. Describe the ordinary steelyard, and explain briefly the principle on which it acts.

22. Explain clearly why the arms of a balance should be equal.
23. Mention some characters of a good balance.
24. A rod of uniform thickness, weighing 5 lbs., rests by its ends on two supports, what is the pressure on each? If the length of the rod is 4 feet, and a weight of 12 lbs. is hung at a distance of 1 foot from one support, find the total pressure on that support.
25. Describe the wheel and axle, and show that it is a modification of the lever.
26. Draw a diagram to represent a simple form of the wheel and axle, and state the conditions of equilibrium.
27. What advantage has the wheel and axle over the ordinary lever?
28. Describe two examples of the application of the wheel and axle.
29. If the handle of a windlass be 2½ feet long, and the diameter of the barrel 8 inches, what force on the handle will be required to support a weight of 840 lbs.?
30. The crank or handle of a windlass is 14 inches long; what must be the diameter of the axle when a man exerting a force of 60 lbs. on the handle raises a load weighing 2 cwt.?
31. The circumference of the barrel of a windlass being 24 inches, through what distance will the power which moves the handle travel in raising the weight 10 feet, the ratio of P to R being as 2 to 11?
32. A capstan with levers 7 feet 6 inches long is worked by six men, each exerting a force of 47 lbs.; the radius of the axle is 1 foot 3 inches, what weight will the men support?
33. What is a pulley? Mention some purposes for which pulleys are employed.
34. What is meant by a fixed, and what by a movable pulley? What advantage is gained by the use of a fixed pulley?
35. Draw a diagram representing a combination of one fixed and one movable pulley, hanging from a fixed beam, and state the relation between the power and resistance in such arrangement, neglecting the weight of the pulleys themselves. If the power is 8 lbs., what will be the total pressure on the beam?
36. Draw a diagram to represent a system of 3 movable pulleys, and state what relation between power and weight will produce equilibrium.
37. In the system of pulleys where the same string is passed round all the pulleys, there are 8 strings connected with the lower block, and the power is 25 lbs.; find the weight.
38. Why is the power only half the weight in a single movable pulley when the strings are parallel? Show that the work done by the power is equal to the work done against the resistance when the weight is raised.
39. What is an inclined plane? Give some illustrations of the use of the inclined plane.
40. How does friction affect the working of the pulley, and of the inclined plane?
41. A cart with its load weighs one ton; with what force must a horse pull to sustain the cart on a road which inclines 1 in 10?
42. The height of an inclined plane is 3 inches, length 18 inches, and power 28 lbs.; what is the weight?
43. A body weighing 100 lbs. is supported on an inclined plane, having an inclination of 1 in 8, partly by friction and partly by a force of 6 lbs. acting up the plane; find the amount of friction.
44. A smooth inclined plane is 8 feet long and 2 feet high; what amount of work is performed in drawing a weight of 100 lbs. up 4 feet of the incline, and what is the least force that will do this work?
45. A heavy box weighing 3 cwt. is being lowered from a cart down a plank, which rests with one end on the cart at a height of 3 feet 6 inches, and the other end on the ground at a distance of 6 feet. What force applied horizontally would prevent the box from falling?

46. A body weighing 14 lbs. rests on a smooth horizontal plane; how great is the reaction exerted by the plane? Will the reaction be more or less when the plane is inclined?
47. Describe some operations in which a wedge is employed. Name some common tools whose action depends on the principle of the wedge.
48. Describe a wedge, and state for what reason it is driven by blows.
49. Show that friction acts advantageously in the case of a wedge.
50. In what respect does the mechanical advantage of the wedge depend upon the angle of its edge? Explain your answer according to the principle of work.
51. Show that the screw is simply a modification of an inclined plane. What are the theoretical conditions of equilibrium in the screw?
52. Of what use is the screw? Show that in some cases its use depends on friction.
53. What is meant by the pitch of a screw? A screw having 12 threads to the inch is turned round 4 times; how far will it have moved in the direction of its length?
54. A screw whose pitch is ¼-inch is turned by a lever 4 feet long; find the power which will support 15 cwt.?
55. What is the relation between the power and the resistance in the case of a screw? If the circumference of the circle described by the power in a screw press is 5 feet, the power 12 lbs., and the distance between the threads 1 inch, what pressure is produced?
56. Mention some practical applications of the screw.
57. A screw having 4 threads to the inch is employed to move a body; how far will the body be moved by each complete revolution of the screw, and what power will be required at the end of a lever 4 feet long to overcome a resistance of 720 lbs.?
58. By what simple experiment could you prove that the direction in which gravity acts is perpendicular to the free surface of a liquid?
59. Describe some experiments to prove that liquids transmit pressure in all directions.
60. What part of a vessel containing a liquid has pressure exerted on it by the liquid? What is the cause of this pressure? Is the pressure the same on all parts of the vessel?
61. The pressure of a liquid at any point is proportional to the depth. Explain this, and mention any facts you know in proof of the statement.
62. If the pressure on a square inch of surface at a depth of 144 feet in a liquid is 1000 lbs., what will be the pressure per square inch at a depth of 216 feet?
63. What is the pressure per square inch at a depth of 110 fathoms in a lake?
64. Why does the pressure of a liquid on the sides of a vessel increase with the depth? What force would be required to pull off a sucker which had been pressed close to the sides of a tank at a depth of 33 feet below the surface of the water?
65. What is the total pressure on the base of a vessel having an area of 1 square foot, and covered with mercury to a depth of 10 inches?
66. Why is it very difficult to move a lock-gate when the water is at unequal heights on the two sides; and why is it comparatively easy when the level is the same on both sides?
67. Explain the principle of the hydrostatic press, and state what practical use is made of it.
68. Describe briefly the construction of Bramah's press. If the area of the small plunger is ¾ of a square inch, and that of the ram 30 square inches, and the arms of the lever handle 2 inches and 2 feet respectively, what force will be required at the end of the lever to support a weight of 5,760 lbs. on the ram?

69. The large piston of a hydrostatic press has an area of 4 square feet, and the smaller piston an area of 8 square inches; when a pressure of 80 lbs. is applied to the smaller piston, what will be the total pressure on the larger piston, and what will be the pressure on each square inch?
70. What is meant by the resultant of two forces? Find the resultant of two forces of 15 and 20 lbs. respectively acting on a point at right angles to each other.
71. Two forces of 15 and 18 lbs. respectively act on a point; find the direction and magnitude of their resultant (*a*) when they act in the same direction, (*b*) in opposite directions, (*c*) at right angles.
72. What is the parallelogram of forces? Apply it to resolve a force of 25 lbs. into two forces making an angle of about 60° with each other.
73. Show that the smaller the angle between two forces the greater the resultant. What is the greatest, and what the least, resultant which two forces of 10 and 11 lbs. respectively can have?
74. Represent by a diagram two forces of 9 and 15 lbs. respectively acting on a point, and making an angle of 45° with each other; and show how to find the direction and magnitude of a third force which would keep the point in equilibrium.
75. A ship is steered due north with a velocity of 5 miles per hour, and the tide is running due west with an equal velocity. In what direction will the ship move, and with what velocity?
76. A man wishes to cross a river, having a strong current, in a boat, to a point exactly opposite to that from which he starts; how will it be necessary for him to steer his boat? Give reasons for your answer.
77. In a stream a mile wide the current flows at the rate of 3 miles an hour from East to West, and a man rows a boat across at a velocity of 4 miles an hour, keeping the boat pointing constantly North and South. Draw a diagram to show the actual course of the boat, and state at what point the boat will touch the opposite bank.
78. Explain how the rudder of a boat acts.
79. A stone is tied to a string and allowed to hang in a rapid stream; show by a diagram the forces acting upon the stone, and the direction which the string will assume.
80. A body rests on an inclined plane; show by a diagram how to resolve its weight into two forces, one urging the body down the plane, and one producing a pressure on the plane.

GEORGE PHILIP AND SON, PRINTERS, CAXTON BUILDINGS, LIVERPOOL.

A

SELECTED LIST

OF

EDUCATIONAL WORKS

PUBLISHED BY

GEORGE PHILIP & SON.

LONDON: 32, FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, SOUTH JOHN STREET,
AND 49 & 51, SOUTH CASTLE STREET.

PHILIPS' SERIES OF SCHOOL ATLASES.

NEW EDITIONS. REVISED & ENLARGED.

PHILIPS' COMPREHENSIVE ATLAS of Ancient and Modern Geography, comprising Forty-two Modern and Eighteen Ancient Maps. Edited by W. HUGHES, F.R.G.S. With Index. *New edition, revised and enlarged.* Imperial 8vo, strongly half-bound, 10s. 6d.

PHILIPS' STUDENT'S ATLAS, containing Forty-eight authentic Maps of the Principal Countries of the World. Edited by W. HUGHES, F.R.G.S. With Index. *New edition, revised and enlarged.* Imperial 8vo, strongly bound in cloth, 7s. 6d.

PHILIPS' SELECT SCHOOL ATLAS, containing Thirty-six authentic Maps of the Principal Countries of the World. Edited by W. HUGHES, F.R.G.S. With Index. *New edition, revised and enlarged.* Imperial 8vo, strongly bound in cloth, 5s.

PHILIPS' INTRODUCTORY SCHOOL ATLAS, containing 24 Maps of the Principal Countries of the World. Edited by WILLIAM HUGHES, F.R.G.S. With Index. *New edition, revised and enlarged.* Imperial 8vo, bound in cloth, 3s. 6d.

PHILIPS' PHYSICAL ATLAS, containing a Series of Maps and Diagrams illustrating the Natural Features, Climates, Various Productions, and Chief Natural Phenomena of the Globe. With Notes. *New and revised edition.* Imperial 8vo, strongly bound in cloth, 5s.

"When we state that the above Atlases form a portion of a list of thirty-two, commencing with a threepenny one containing sixteen maps, and including the excellent guinea 'Historical Atlas,' and the scarcely less valuable 'Training College Atlas,' we indicate the spirit with which Messrs. Philip have taken up this branch of educational publications. A cursory examination of any of the series will satisfy any one of the excellent manner in which they are got up. They are clear, beautifully engraved, and carefully coloured. The names are introduced with great judgment—free from the common fault of overcrowding. A copious index accompanies each of the above atlases, with the latitude and longitude of each place mentioned. The books are also strongly bound—no slight advantage in regard to an atlas."—*Practical Teacher.*

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE STREET.

PHILIPS' SERIES OF MAP-DRAWING BOOKS

WITH INSTRUCTIONS

By J. FRANCON WILLIAMS, F.R.G.S.

The present series of "Map-Drawing Books" are unique both in plan and object, and afford the most valuable aid to the attainment of definite and accurate geographical knowledge. Each book contains (1) a complete Full-Colored Map, so attached as to be always available for copying; (2) an Uncolored Map without the names, which the pupil can fill in as an introductory exercise; (3) an Outline Map, with coast-line and lines of latitude and longitude only—the pupil being required to fill in the mountains, rivers, towns, &c., with the names; (4) Map with lines of latitude and longitude only; (5) a blank sheet, with border of map only. These books, therefore, provide for *repeated practice* in drawing each map—which cannot fail to impress the principal features, both physical and political, more firmly on the memory than by merely referring to an atlas.

Imperial 4to Series—13 in. by 11 in.—price 6d. each.

LIST OF THE SERIES.

1 **Europe**
2 **England and Wales**
3 **Scotland**
4 **Ireland**
5 **France**
6 **Holland**
7 **Sweden and Norway**
8 **Russia in Europe**
9 **German Empire**
10 **Austro-Hungarian Empire**
11 **Spain and Portugal**
12 **Switzerland**
13 **Italy**
14 **Turkey in Europe and Greece**
15 **Asia**
16 **India**
17 **Australia**
18 **New Zealand**
19 **Africa**
20 **North America**
21 **United States**
22 **Canada**
23 **South America**
24 **Palestine**

Crown 4to Series—10 in. by 8 in.—price 3d. each.

LIST OF THE SERIES.

1 **Europe**
2 **England**
3 **Scotland**
4 **Ireland**
5 **France**
6 **Holland and Belgium**
7 **Switzerland**
8 **German Empire**
9 **Austro-Hungarian Empire**
10 **Sweden and Norway, with Denmark**
11 **Russia in Europe**
12 **Turkey in Europe & Greece**
13 **Italy**
14 **Spain and Portugal**
15 **Asia**
16 **India**
17 **Africa**
18 **North America**
19 **Canada**
20 **United States of North America**
21 **South America**
22 **Australia**
23 **New Zealand**
24 **Palestine**

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE ST.

JUST PUBLISHED,

Foolscap 8vo, handsomely bound in cloth, price 2/6.

GEOGRAPHY OF THE OCEANS.

PHYSICAL, HISTORICAL, & DESCRIPTIVE.

With Contour-Map of the Ocean; Map showing the principal Divisions of the Ocean; Continental and Oceanic Hemispheres; Maps of the Atlantic, Pacific, Indian, Arctic, and Antarctic Oceans; Charts of Co-tides and Ocean-Currents; Approximate Sections; Illustrations of the Tides, Oceanic Circulation, &c., &c.

By J. FRANCON WILLIAMS, F.R.G.S.

⁂ The above unique work forms a complete *resumé* of the "Geography" of the Oceans, and is specially designed for the use of Students preparing for Examination. The information throughout has been brought down to the latest date, and the greatest care has been taken to include not only the main facts, but also every *essential* detail. Special attention is directed to the clear and systematic arrangement of the text, and the accompanying Maps and Illustrations. This work is also admirably adapted for use in Schools as a

Supplementary Geographical Text Book.

"This is a most exhaustive treatise on the Geography of the Oceans. . . . It is an admirable work. We know nothing better, whether for the Schoolroom or the advanced student. . . . All the latest authorities have been drawn upon, and the author has made an excellent use of his materials."—*Educational News.*

" . . . The best arranged and best written work of the kind which we can remember to have seen for a considerabe period. . . ."—*Shipping and Mercantile Gazette.*

"Specially suitable as a reading book for advanced classes in schools."—*Bristol Mercury.*

"Students of this important branch of geography will find the information well-arranged, and of the most recent character."—*The Bookseller.*

"Cannot fail to be of the greatest use to students. . . The author has done his work admirably. . . . A most useful and valuable book."—*Eastern Morning News.*

"Clear in statement, accurate in matter, and well calculated to give much needed assistance to students of Geography and Physiography."—*The Principal, Training College, York.*

"Successful in every way. The book is not only valuable for the extent and accuracy of its information, but pleasant reading withal. . . . The work is thoroughly well done, and cannot fail to be highly useful to readers of all kinds, as well as to geographical students. We strongly recommend the book."—*Practical Teacher.*

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE ST.

JUST PUBLISHED,
Crown 8vo, 380 pp., strongly bound in cloth, price 3s. 6d.

CLASS-BOOK OF MODERN GEOGRAPHY.

WITH EXAMINATION QUESTIONS

BY WILLIAM HUGHES, F.R.G.S.,
Late Professor of Geography in King's College, London.

New Edition. Revised and Enlarged
BY J. FRANCON WILLIAMS, F.R.G.S.,
Author of "The Geography of the Oceans," &c.

☞ **In the present edition the information throughout has been thoroughly Revised and considerably Extended. Every recent Discovery and Political Change is carefully noticed. The work is admitted to be by far the best Geographical Text-Book yet published.**

"This excellent text-book of Geography has been revised and brought down to the present year by Mr. J. Francon Williams, whose work has been done most satisfactorily in all respects. The name of William Hughes is, of itself, a passport to the accuracy and merits of any book on Geography, and there is no reason why the work before us should not take the place among the first, if not of itself the first, of geographical class-books. In addition to unfailing accuracy, this book possesses some unusually meritorious features. Among these are a set of the best geographical questions to be met with. Next we may notice the value of the explanations and derivations of geographical names. These are in most cases traced to original sources, instead of being merely copied from second-hand or questionable authorities. . . . . The amount of information contained in the 380 pages of this admirable class-book is remarkable. And when we state that this is not given in a bald way, but with a fair amount of explanation, and in a readable style, we say nothing more than the book merits, but which few books of the same nature and design possess."—*Practical Teacher.*

"Cannot fail to obtain a wide circulation."—D. WILLIAMS, Esq., *Training College, Swansea.*

"Admirably adapted for senior classes."—G. THOM, Esq., *Dollar, N.B.*

"Contains a combination of excellences that must render it very valuable." —DR. MORRISON, *Glasgow Academy.*

"I have no hesitation in saying that it is a judicious and accurate compilation of geographical knowledge, and well adapted for school use."—DR. HAIG BROWN, *Charterhouse School.*

"Contains a vast amount of fresh and interesting information. I think the examination questions a very valuable addition."—DR. PARKER, *Belfast.*

"The new edition by Mr. Williams brings it up accurately to the present state of our knowledge. . . . It is in every respect admirably adapted for the purposes of tuition in our higher class schools."—DR. MILLER, *Perth.*

"A work which, for clearness, fulness, and excellence of arrangement, can hardly be surpassed."—*Literary World.*

"I have examined it carefully, and I know no text-book on the same subject that can be compared to it. I at once introduced it as a text-book for the pupil teachers."—T. W. WALLACE, Esq., *High School, Inverness.*

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE STREET.

IMPORTANT GEOGRAPHICAL WORKS.

BY WILLIAM HUGHES, F.R.G.S.,

Late Professor of Geography in King's College, London.

CLASS-BOOK OF MODERN GEOGRAPHY, with Examination Questions. The Examination Questions are drawn from the result of much experience in tuition on the part of the Author, and will be found to add considerably to the value of the work as a class-book for popular school use. New Edition, revised and greatly enlarged, crown 8vo, cloth, 3s. 6d.

⁂ "Philips' Comprehensive School Atlas" is designed to accompany this work.

ELEMENTARY CLASS-BOOK OF MODERN GEOGRAPHY, abridged from the larger class-book, and designed for the use of less advanced pupils. New and revised edition, foolscap 8vo, 1s. 6d.

⁂ "Philips' Atlas for Beginners" is designed to accompany this work.

CLASS-BOOK OF PHYSICAL GEOGRAPHY, with numerous Diagrams and Examination Questions, by William Hughes, F.R.G.S. This volume has been prepared for popular school use, and exhibits, in a clear and methodical manner, the principal facts respecting the Natural Features, Productions, and Phenomena of the Earth. New edition, entirely re-written and extended, with Maps and Illustrations. Crown 8vo, cloth, 3s. 6d.

⁂ "Philips' School Atlas of Physical Geography" is designed to accompany this work.

ELEMENTARY CLASS-BOOK OF PHYSICAL GEOGRAPHY, with Diagrams, by William Hughes, F.R.G.S. Intended as a Companion Text-Book to "Philips' Physical Atlas for Beginners." Foolscap 8vo, cloth, 1s.

BY W. LAWSON, ST. MARK'S COLLEGE, CHELSEA.

OUTLINES OF GEOGRAPHY, for Schools and Colleges. *New edition, entirely re-written and extended.* Foolscap 8vo, cloth, 2s. 6d.

This book may also be had in Parts, viz.:—

I.—The British Islands, 6d.
II.—The British Colonies, 6d.
III.—Europe, 9d.
IV.—Asia, Africa, America, 1s.

THE GEOGRAPHY OF RIVER SYSTEMS. *New and revised edition.* Foolscap 8vo, cloth, 1s.

THE GEOGRAPHY OF COAST LINES. *New and revised edition.* Foolscap 8vo, cloth, 1s.

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE STREET.

NEW AND REVISED EDITION.

GEOGRAPHY

OF THE

BRITISH COLONIES

AND FOREIGN POSSESSIONS.

For the use of Candidates preparing for Examination, by the Rev. JOHN P. FAUNTHORPE, M.A., F.R.G.S., Principal of Whitelands College, Chelsea. Crown 8vo, cloth, price 2s, 6d.

"Nothing seems to be lacking to entitle it to be described as a complete manual of instruction on our colonial possessions. Historical and geographical facts are skilfully blended. . . . reflects credit on the industry of the author. We confidently recommend the work to students and others who require full and accurate information on this branch of geographical study."—*The Schoolmaster.*

"Throughout the book, historical matter is deftly interwoven with geographical information. The work is a speciality in geography, and is thoroughly well done."—*School Board Chronicle.*

"Full of valuable and interesting information, both as to the history and geography of our possessions; and all is so clearly arranged as to give the greatest possible assistance in study."—*Scholastic World.*

The above work is designed as a Handbook to

PHILIPS' ATLAS OF THE BRITISH EMPIRE THROUGHOUT THE WORLD. A Series of Twenty-three Maps, with explanatory and statistical Notes, by John Bartholomew, F.R.G.S. Imperial 8vo, bound in cloth, 3s. 6d.

"This is an excellent atlas. . . The charts are clear and distinct, and the information conveyed by the 'Notes' of value and interest."—*Glasgow Herald.*

"Of the maps we cannot speak too highly. They are executed in the first style of art, and are a credit to the firm which has issued them."—*Educational Times.*

"The maps are minutely accurate, and so clearly printed as to be easy of reference."—*The Schoolmaster.*

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE ST.

ENGLISH HISTORY FOR SCHOOLS

BY

JOHN CURNOW, B.A., LL.D.

Crown 8vo, cloth, price 2s. 6d.

⁂ *This work is not only well suited for use as an ordinary text-book of English History, but is also admirably adapted as a*

HISTORICAL READING BOOK.

OPINIONS OF THE PRESS.

"This History of England has been written with the object of producing a historical picture, in which the lights and shades are so disposed as to throw up clearly the prominent events and characters. . . . . Merit of great clearness, both of arrangement and style."—*Educational Times.*

"In his treatment of those sovereigns and personages who form, as it were, the personal embodiment of the great movements of the age in which they lived, Mr. Curnow has exhibited a rare discrimination and literary skill. . . . Mr. Curnow's History shows accurate thought and learning, and the style of writing evinces a refined taste, combined with wonderful freshness and vigour."—*Christchurch Press.*

"The events are told in so pleasant and accurate a manner, that the attention ought never to flag."—*School Guardian.*

"The book . . . is a perfect marvel of execution. It is a positive pleasure to turn from the closely-packed pages of the Student's "Hume" to the beautiful paragraphs of this work."—*Lyttleton Times.*

MANUAL OF BRITISH HISTORICAL GEOGRAPHY, comprising the most important events occurring at every site of English History. For the use of Students preparing for Examination By W. J. C. CRAWLEY. Foolscap 8vo, cloth, 2s.

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51, SOUTH CASTLE STREET.

DAVIES' HISTORICAL MANUALS.

Designed for the use of Pupils preparing for the University Local and other Examinations.

"**Evidently the result of laborious research and extensive scholarship.**"
—***The Headmaster, Archbishop Tenison's School, London.***

Manual of the History of England, from the death of Edward the Confessor to the death of King John (1066-1216). Foolscap 8vo, 2s.

Manual of the History of England, from the accession of Henry III. to the death of Richard III. (1216-1485). Foolscap 8vo, cloth, 2s.

Manual of the History and Literature of the Tudor Period (1485-1603) to the accession of James VI. of Scotland. 256 pp. Foolscap 8vo, cloth, 2s.

Manual of the History and Literature of the Stuart Period, to the accession of William III. and Mary II. (1603-1689). 160 pp. Foolscap 8vo, cloth, 1s. 6d.

Manual of the History of England, from the reign of Charles I. to the end of the Commonwealth (1640-1660). Foolscap 8vo, cloth, 1s. 6d.

Manual of the History of England, from the restoration of Charles II. to the Revolution (1660-1688). Foolscap 8vo, cloth, 2s.

Manual of the History of England, from the accession of James I. to the Battle of the Boyne (1603-1690). Foolscap 8vo, cloth, 2s.

Manual of the History of England, from the Revolution of 1688 to the death of Queen Anne, 1714. Foolscap 8vo, cloth, 1s. 6d.

Manual of the History of England, from the accession of William III. to the accession of George III. (1689-1760), and the Outlines of English Literature during the same period. Foolscap 8vo, cloth, 2s. 6d.

Manual of the History of England, from the accession of George III. to the Battle of Waterloo (1760-1815 A.D.) Foolscap 8vo, cloth, 2s. 6d.

Manual of English Literature (1760-1815), **from the accession of George III. to the Battle of Waterloo.** Foolscap 8vo, cloth, 2s.

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE ST.

PHILIPS' SERIES OF SCRIPTURE MANUALS.

Designed for the use of Pupils preparing for the
OXFORD AND CAMBRIDGE LOCAL EXAMINATIONS.

BY THE LATE JAMES DAVIES.

Uniformly Printed on Foolscap 8vo, bound in cloth.

Notes on Genesis, 1s.
Notes on Exodus, 1s.
Notes on Joshua, 1s.
Notes on Judges, 1s.
Notes on I. Samuel, 1s.
Notes on II. Samuel, 1s. 6d.
Notes on I. Kings, 1s. 6d.
Notes on II. Kings, 1s. 6d.
Notes on Ezra, 1s.
Notes on St. Matthew's Gospel, 2s.
Notes on St. Mark's Gospel, 1s.
Notes on St. Luke's Gospel, 1s. 6d.
Notes on St. John's Gospel, 2s. 6d.
Notes on Acts of the Apostles, 1s 6d.
Manual of the Book of Common Prayer, 2s.
Manual of the Church Catechism, 1s.

BY THE REV. HENRY LINTON, M.A.

THE BOOK OF NEHEMIAH. With Explanatory Notes and Appendices. Foolscap 8vo, cloth, 1s. 6d.

THE BOOK OF JEREMIAH. Part I. Historical Chapters, with Notes and Appendices. Foolscap 8vo, cloth, 2s.

THE BOOK OF GENESIS. With Explanatory Notes and Appendices. Foolscap 8vo, cloth, 2s.

I. CORINTHIANS. With Explanatory Notes, Appendices, and Map. Foolscap 8vo, cloth, 1s. 6d.

Opinions of the Press on Philips' Series of Scripture Manuals.

"Very carefully done, and contains a large amount of information."—*School Guardian.*

"These Manuals have our heartiest commendation."—*Schoolmaster.*

"The notes are very full, and the matter is so arranged as to render great assistance to the student."—*Scholastic World.*

"Good in idea and plan, and gives evidence of careful and honest work."—*Educational Record.*

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE ST.

ADVANCED ARITHMETIC

FOR SCHOOLS AND COLLEGES.

By THOMAS W. PIPER, late Normal Master and Lecturer in the National Society's Training College, Battersea. Crown 8vo, cloth, price 3s. 6d.

"The chief excellence of this Arithmetic is its completeness. It is full at all points. It is never wanting either in detail or compass. But while the author is strong on the demonstration of reasons in arithmetic, he is wise enough at the same time to be strong on rules. . . . In his appendix Mr. Piper does not content himself with giving selections of examination papers, but first presents five-and-twenty pages of solutions of problems."—*School Board Chronicle.*

BY THE SAME AUTHOR,

ELEMENTARY TREATISE ON ARITHMETIC, for Schools and Colleges. Crown 8vo, cloth, 1s. 6d.

"The author's endeavour to make the study of Arithmetic what it ought to be,—a thorough mental training,—is apparent on every page."—*Teacher's Assistant.*

MENTAL ARITHMETIC, containing Rules and Exercises founded on Examination Papers. *New and enlarged edition.* Foolscap 8vo, cloth, 2s.

"Has had a deserved success."—*School Guardian.*

"Contains an excellent collection of examples."—*Scholastic World.*

INTRODUCTORY MENTAL ARITHMETIC, being a short Exposition of the uses of Mental Arithmetic, with illustrative examples and a great number of New and Original Exercises. Foolscap 8vo, stiff cover, 6d.

"A valuable little work."—*Scholastic World.*

LONDON: GEORGE PHILIP & SON, 32, FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 AND 51, SOUTH CASTLE ST.

Crown 8vo, cloth, price 3s. 6d.

THE ELEMENTS OF EUCLID

FOR SCHOOLS AND COLLEGES

By James Martin, Head Master of the Endowed School,
Wedgewood Institute, Burslem.

In directing the attention of Teachers and Students to this work, the Publishers desire to point out its leading characteristics:—

(1.) **In all the propositions, a clear line of demarcation is drawn between the Construction and the Proof or Demonstration.**

(2.) **By a typographical expedient the several steps in the reasoning are clearly shewn.**

(3.) **In describing the figures, those parts which are given in the enunciation are represented by dark lines, and those which are added in the course of the demonstration by dotted lines.**

(4.) **In all cases the figure has been repeated wherever it was found necessary.**

The present edition of "Euclid's Elements" contains the First Six, and those parts of the Eleventh and Twelfth Books which are usually read at the Universities, together with a valuable selection of Geometrical Problems for Solution, which cannot fail to be of very great service to every student of Geometry.

BY THE SAME AUTHOR.

THE ELEMENTS OF EUCLID, with a Selection of Geometrical Problems for Solution. Crown 8vo, Book I., cloth, 1s.; Books I. and II., 1s. 6d.

***** Specimen Copies of the above work will be forwarded to Teachers on receipt of half the published price.**

London: GEORGE PHILIP & SON, 32 Fleet Street, E.C.
Liverpool: Caxton Buildings, and 49 & 51 South Castle St.

A GRADUATED COURSE

OF PROBLEMS IN

PRACTICAL PLANE & SOLID GEOMETRY,

By JAMES MARTIN. Crown 8vo, Cloth, price 3s. 6d.

The Publishers have much pleasure in drawing the attention of Teachers and Students to this work, which is admittedly the best of the kind yet published. Special attention is directed to the following characteristic features of the work:—

1. **In the Introductory section full particulars are given of the various drawing instruments, and the manner of using them, together with general hints on Drawing.**
2. **The Problems in "Plane Geometry," though exceedingly numerous, are classified in sections.**
3. **The Diagrams are engraved with extreme care, and for the sake of clearness, three kinds of lines are used, viz:—**
   - (1.) Thin lines, representing those which are given.
   - (2.) Dotted lines, showing those used in the construction of the figure.
   - (3.) Thick lines, representing the solution of the problem.
4. **By a typographical expedient, the two cardinal ideas, viz.—what is given, and what is to be done—are clearly shown in the enunciation.**
5. **The Problems in "Solid Geometry" are also carefully graduated, and arranged in sections.**
6. **An exhaustive section on the derivation of Geometrical problems.**
7. **A complete Index of all the Problems in both parts of the work.**

Crown 8*vo, stiff cover, price* 3*d.*

FIRST GRADE PRACTICAL GEOMETRY,

BY DAVID BAIN, F.R.G.S.

Containing—(*a*) all the necessary problems fully worked out; (*b*) test exercises on the same; (*c*) a series of graduated exercises for copying; and (*d*) all the First Grade Examination Papers given by the Department during the last three years.

"The book is well got up, the diagrams are clear, and the explanation of each problem is short and to the point."—*Schoolmaster.*

"Diagrams very clearly drawn."—*Scholastic World.*

"Instructions on the working of the problems are clear and precise. The pupil who masters the problem and test exercises given in this little book can scarcely fail to pass any examination in elementary practical geometry." *School Guardian.*

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE ST.

Crown 8vo., cloth, with numerous illustrations, price 2s. 6d.

ACOUSTICS, LIGHT, AND HEAT,

INTENDED AS AN

INTRODUCTION TO THE STUDY OF PHYSICAL SCIENCE.

Adapted to the requirements of the Science and Art Department, by T. W. PIPER, late Normal Master, St. John's College, Battersea.

⁂ *The plan of the work is different from that of any manual yet issued. It is not a dry epitome: it is an attempt to set forth the great principles of this branch of Physical Science in clear and familiar language.*

"Mr. Piper's book has special features, which will readily commend themselves to teachers. The chief of these features is the grouping of similar phenomena, treating the three cognate sciences side by side, and bringing into prominent notice their many points of agreement, and, of course, also their differences. Thus, after a preliminary chapter on the atmosphere, the medium through which sound, light, and heat are usually received, vibratory motion of air, and other forms of matter and ether are discussed. In the same way are grouped the laws of direction of sound and light waves, with their similar heat waves, the conservation and energy of matter, and the phenomena of sensation. We cannot speak too highly in praise of this arrangement. The book is well filled, and the illustrations are numerous and appropriate."—*Schoolmaster.*

"Mr. Piper's *Acoustics, Light, and Heat,* deserves the careful attention of teachers of science classes. It is a great improvement on the manuals in common use. . . We may state that the "Law of Inverse Squares" is more clearly stated and illustrated than in any book we have before seen. . . The work is so well done, that the non-scientific reader can hardly fail to derive much pleasure from it, while to the science teacher it will be invaluable."—*School Guardian.*

"By far the best and clearest elementary treatise on the subject."—*The Principal, Training College, York.*

"Written in a style peculiarly engaging and interesting."—*Oldham Evening Chronicle.*

"Mr. Piper's book is the result of hard work and competent knowledge, both of the subject, and, what is all too frequently ignored in our school books, how to teach it."—*The Principal, Training College, Whitelands.*

. . . "Deserving of praise, and very suitable for the use of science classes."—*The Principal, Training College, Ripon.*

"A useful introduction to the study of Physical Science. . . Practical, and in many instances, novel illustrations of the principles are given."—*The Bookseller.*

"Very complete hand-book . . commences with facts that are almost self-evident, stated in the simplest terms, and gradually, by slow but consecutive stages, passes to the more complex phenomena."—*Iron.*

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE ST.

New and enlarged edition. Crown 8vo, cloth, price 2s. 6d.

CLASS-BOOK OF INORGANIC CHEMISTRY.

BY D. MORRIS, B.A.,

Late Teacher of Chemistry in Liverpool College.

⁂ *The changes made in the present edition are such as greatly enhance the value of the work as a text-book for students preparing for examination.*

"A capital hand-book for students reading for the University Examinations."—*School Guardian.*

"To the student in chemistry who wishes to get possession of the greatest possible number of facts in a given time, the book will be eminently useful."—*British Mail.*

"We can heartily recommend this manual to those preparing for the 'Middle-Class Examinations,' and the London Matriculation. . . . . Brimful of accurate information, brought up to the latest date."—*The Teacher.*

Crown 8vo, cloth, price 1s. 6d.

CLASS-BOOK OF ELEMENTARY MECHANICS

AN INTRODUCTION TO NATURAL PHILOSOPHY.

BY W. HEWITT, B.SC.,

Science Demonstrator for the Liverpool School Board.

"It is very pleasant to meet with a book so fresh, so thorough, and so simple. . . . The book before us is the outcome of the thought of a practical teacher, and of a clear and logical mind. The questions at the end of each chapter are extremely well chosen. The good taste shewn in the get-up of the book, the clearness of the type, &c., are worthy of the excellent manner in which Mr. Hewitt has treated his subject."—*Schoolmaster.*

"The exercises are a very praiseworthy part of Mr. Hewitt's work. . . Common incidents, which through their very commonness are deemed insignificant, are here made the vehicle of most important instruction."—*Practical Teacher.*

"We have seldom met with a really elementary book which at once combined to so great a degree simplicity of language, accuracy of description, and sound science."—*Nature.*

Extra foolscap 8vo, cloth, price 1s. 6d.

CLASS-BOOK OF ELEMENTARY GEOLOGY.

BY F. WOLLASTON HUTTON, F.G.S.

⁂ ***This little work is a systematic and complete resumé of the subject, and is specially adapted for beginners.***

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51, SOUTH CASTLE ST.

Medium Folio, bound in cloth, 18s.

THE TRAINING COLLEGE ATLAS.

A Series of Twenty-four Maps, illustrating the Physical and Political Geography of the Chief Countries of the World. Originally designed and drawn by WILLIAM HUGHES, F.R.G.S.

NEW AND ENLARGED EDITION, EXTENDED AND COMPLETED,

BY E. G. RAVENSTEIN, F.R.G.S.

⁂ The "Training College Atlas" is a work altogether distinct in character from any previously published. In its production the Publishers have aimed at combining the fullest and most recent physical and political information with the greatest possible clearness and distinctness, and it is believed that the new Atlas will be found to possess both these important features in a higher degree than any yet issued.

"It would be difficult, if not impossible, to supersede this really handsome and serviceable Atlas, which we think the best of the many successful efforts which Messrs. Philip & Son have yet put forth as geographical publishers."—*The School and University Magazine.*

"All the recent political changes in different parts of the world are carefully represented."—*School Guardian.*

"It strikes me as an *excellent* Atlas, not only for Training Colleges, but for general purposes."—*One of H.M. Senior Inspectors of Schools.*

"I cannot speak too highly of this Atlas. It will make the study of geography a real pleasure."—*The Lecturer on Geography, Training College, Cheltenham.*

"It seems to me to combine the qualities of clearness, fulness, neatness, and accuracy."—*The Principal, Training College, York.*

"I am not aware that I have ever seen an Atlas to compare with it."—*One of H.M. Inspectors of Schools.*

"I may state that I have carefully examined all the maps in it, and consider them in every respect excellent. The common error of loading the maps with names of insignificant villages, useful to nobody, has been avoided; at the same time, all places which are of interest are clearly marked. The maps are most beautifully drawn and coloured. The physical maps are most valuable."—*One of H.M. Senior Inspecters of Schools.*

⁂ Complete Lists of Maps, Atlases, &c., on application.

LONDON: GEORGE PHILIP & SON, 32 FLEET STREET, E.C.
LIVERPOOL: CAXTON BUILDINGS, AND 49 & 51 SOUTH CASTLE ST.